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Linearization of Thermo-Viscous Fluid in a Porous Slab Bounded Between two Fixed Permeable Horizontal Parallel Plates in the Absence of Thermo-Mechanical Interaction Coefficient

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Abstract— In this paper the problem of the steady flow of a second order thermo-viscous fluid through a porous slab bounded between two fixed permeable parallel plates is examined. There is a constant injection at one plate and equal suction at the other plate. The two plates are kept at two different temperatures and the flow is generated by a constant pressure gradient. The solutions of governing equations of the flow with appropriate boundary conditions have been obtained analytically.

Keywords- Darcy's Porosity Parameter; Suction/Injection Parameter; Strain Thermal Conductivity Coefficient.

I. INTRODUCTION

Considerable interest has been evinced in the recent years on the study of thermo-viscous flows through porous media because of its natural occurrence and its importance in industrial geophysical and medical applications. The flow of oils through porous rocks, the extraction of energy from geo-thermal regions , the filtration of solids from liquids , the flow of liquids through ion-exchange beds, cleaning of oil-spills are some of the areas in which flows through porous media are noticed. In the physical world, the investigation of the flow of thermo-viscous fluid through a porous medium has become an important topic due to the recovery of crude oil from the pores of reservoir rocks.

Henry Darcy observed that , the discharge rate of the fluid percolating in a porous medium is proportional to the hydraulic head and inversely as the distance between the inlet and outlet i.e. proportional to the pressure gradient. Darcy , based on the findings of a large number of flows through porous media, proposed the empirical law known as Darcy's law

$$Q = -\frac{k^*}{\mu} A.\nabla P$$

where Q is the total discharge of the fluid, k^* is the permeability of the medium, A is the cross-sectional area to flow the fluid, μ is the viscosity of the fluid and ∇P is the pressure gradient in the direction of the fluid flow. Dividing both sides of the above equation by the area then the above equation becomes

$$q = -\frac{k^*}{\mu} \nabla P$$

where q is known as Darcy's fluid flux and we know that the fluid velocity(u) is proportional to the fluid flux (q) by the porosity(k^*) , then

$$\nabla P = -\frac{\mu}{k^*} u$$

The negative sign indicates that fluids flows from the region of high pressure to low pressure.

Koh and Eringin(1963) introduced the concept of thermo-viscous fluids which reflect the interaction between thermal and mechanical responses in fluids in motion due to external influences. For such a class of fluids, the stress-tensor 't' and heat flux bivector 'h' are postulated as polynomial functions of the kinematic tensor, viz., the rate of deformation tensor 'd':

$$d_{ij} = (u_{i,j} + u_{j,i})/2$$

and thermal gradient bivector 'b'

$$b_{ij} = \in_{ijk} \theta_k$$

where u_i is the i^{th} component of velocity and θ is the temperature field.

A second order theory of thermo-viscous fluids is characterized by the pair of thermo-mechanical constitutive relations:

$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd)$ and $h = \beta_1 b + \beta_3 (bd + db)$

with the constitutive parameters α_i , β_i being polynomials in the invariants of d and b in which the coefficients depend on density(ρ) and temperature(θ) only. The fluid is Stokesian when the stress tensor depends only on the rate of deformation tensor and Fourier-heat-conducting when the heat flux bivector depends only on the temperature gradient-vector, the constitutive coefficients α_1 and α_3 may be identified as the fluid pressure and coefficient of viscosity respectively and α_5 as that of cross-viscosity.

The problem of thermo-viscous fluid flow between two non permeable fixed parallel plates was examined by Pattabhi Ramacharyulu and Nageswar Rao(1969). The flow of thermo-viscous fluid between two parallel plates in relative motion was examined earlier by Pattabhi Ramacharyulu and Anuradha(2006). Nageswar Rao and Srinivas Joshi(2010) investigated the steady flow of thermo-viscous fluid between two parallel porous plates in relative motion. Some visco-metric flows of thermo-viscous fluids was studied by Kelly(1965). The problems on visco-elastic fluid flows was examined by Longlois(1963) and Rivlin(1954). The works of Bear(1970), Beaver and Joseph(1967), Preziosi and Farina(2002) and Yamamoto and Yoshida(1974) presents the flows through the porous medium.

In all these papers the effects of various material parameters on the flows of thermo-viscous fluids have been studied; the present paper deals to study the effects of various material parameters such as porosity parameter, suction/injection parameter and strain thermal conductivity parameter in the absence of thermo-mechanical stress interaction coefficient on the flows through a porous medium.

II. MATHEMATICAL FORMULATION

Consider the steady flow of a second order thermo-viscous fluid through a porous medium bounded between two fixed permeable parallel plates. The flow is generated by a constant pressure gradient in a direction parallel to the plates. Further, the plates are assumed to be permeable allowing a constant injection at the lower plate and equal suction at the upper plate. Let v_0 be the constant Suction /injection velocity.

With reference to a coordinate system O(X,Y,Z) with origin on one of the plates, the X-axis in the direction of the fluid flow, Y-axis perpendicular to the plates. The plates are represented by $y=0$ and $y=h$. The two plates are maintained at constant temperatures θ_0 and θ_1 respectively.

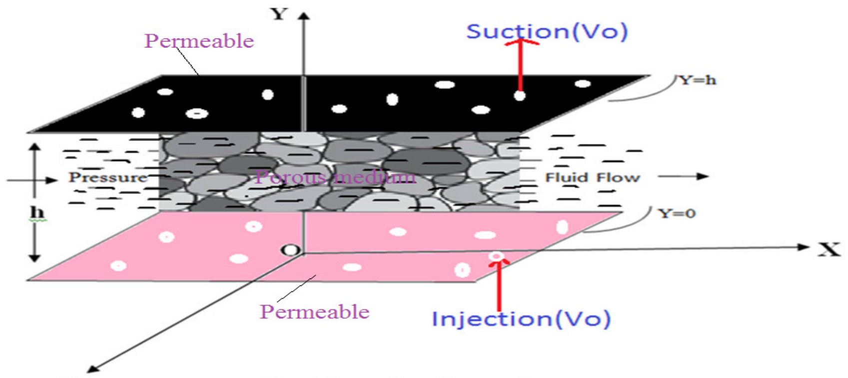


Fig. Flow Configuration

Let the steady flow between the two permeable parallel plates is characterized by the velocity field [$u(y), v_0$, 0] and temperature field $\theta(y)$. This choice of the velocity evidently satisfies the continuity equation.

The basic equations characterizing the flow are the following:
Equation of motion in X-direction :

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \alpha_6 \frac{\partial \theta}{\partial x}\frac{\partial^2 \theta}{\partial y^2} - \frac{\mu}{k^*}u \qquad (1)$$

Equation of motion in Y-direction :

$$0 = \mu_c \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 + \rho f_y \qquad (2)$$

in the Z- direction :

$$0 = \alpha_8 \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y}\right) + \rho f_z \qquad (3)$$

and the energy equation:

$$\rho c\left(u\frac{\partial \theta}{\partial x} + v_0 \frac{\partial \theta}{\partial y}\right) = \mu\left(\frac{\partial u}{\partial y}\right)^2 - \alpha_6 \frac{\partial \theta}{\partial x}\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} + k\frac{\partial^2 \theta}{\partial y^2} + \beta_3 \frac{\partial \theta}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k^*}u^2 \qquad (4)$$

The boundary conditions are :

$u = 0$, $\theta = \theta_0$ at $y = 0$ and

$u=0$, $\theta=\theta_1$ at $y = h$ (5)

III. SOLUTION OF THE PROBLEM

The following non-dimensional quantities are introduced to convert the above basic equations to the non-dimensional form.

$$Y=\frac{y}{h},\ U=\frac{\rho h}{\mu}u,\ \ u_0=\frac{\mu}{\rho h},\ U_0=\frac{\rho h}{\mu}u_0,\ T=\frac{\theta-\theta_0}{\theta_1-\theta_0},\ C_2=\left(\frac{h}{\theta_1-\theta_0}\right)\frac{\partial\theta}{\partial x},\ S=\frac{h^2}{k^*},\ V_0=\frac{\rho h}{\mu}v_0,$$

$$p_r=\frac{\mu c}{k},\ b_3=\frac{\beta_3}{\rho h^2 c},\ a_6=\frac{\alpha_6\rho(\theta_1-\theta_0)^2}{\mu^2}\ \text{ and }\ a_1=\frac{\mu^2}{\rho h^2 c(\theta_1-\theta_0)}.$$

where $C_2\left[=\frac{h}{\theta_1-\theta_0}\left(\frac{\partial\theta}{\partial x}\right)\right]$ are non-dimensional temperature gradient which is assumed to be constant. Further , S is the non-dimensional Darcy's porosity parameter and V_0 is the non-dimensional Suction/injection parameter.

Further, the interaction between the mechanical stress and thermal gradients (characterized by the coefficient a_6) is neglected (i.e. the terms containing the coefficient of a_6 in momentum and energy balance equation are neglected). The equation of motion in the X-direction becomes linear by neglecting the thermo-mechanical stress interaction coefficient(a_6) . The variations of the velocity and the temperature distributions with different values of material parameters such as Strain thermal conductivity coefficient(b_3), Darcy's porosity parameter(S) and Suction/injection parameter (V_0) have been discussed with the help of illustrations.

The equation of momentum and energy in terms of the above non-dimensional quantities now reduce to the following:
Equation of motion in X-direction:

$$V_0\frac{dU}{dY}=-C_1+\frac{d^2U}{dY^2}-SU \tag{6}$$

and the energy equation :

$$UC_2+V_0\frac{dT}{dY}=a_1\left[\left(\frac{dU}{dY}\right)^2-\mathrm{SU}^2\right]+b_3C_2\frac{d^2U}{dY^2}+\frac{1}{p_r}\frac{d^2T}{dY^2} \tag{7}$$

together with the boundary conditions :

$$U(0)=0\quad,\quad U(1)=0 \tag{8}$$

and

$$T(0)=0\quad,\quad T(1)=1 \tag{9}$$

The equation (6) and the boundary conditions in (8) yields the velocity distribution

$$U(Y)=\frac{C_1}{S\sinh m_2}\left\{e^{m_1(Y-1)}\sinh m_2Y-e^{m_1Y}\sinh m_2(Y-1)-\sinh m_2\right\} \tag{10}$$

The equations (7), (10) and the boundary conditions in (9) yields the temperature distribution

$$T(Y)=\frac{YC_1(a_1C_1-C_2)}{mS}+\left[\frac{e^{mY}-1}{e^m-1}\right]\left(1-\frac{C_1(a_1C_1-C_2)}{mS}\right)$$
$$-\frac{a_1AB(m_1^2-m_2^2-S)}{m_1(2m_1-m)(e^m-1)}\left\{e^{2m_1}(1-e^{mY})+(e^{mY}-e^m)-e^{2m_1Y}(1-e^m)\right\}$$
$$+\frac{A\left(C_2-2a_1C_1-b_3C_2(m_1+m_2)^2\right)}{(m_1+m_2)(m_1+m_2-m)(e^m-1)}\begin{Bmatrix}e^{m_1+m_2}(1-e^{mY})+(e^{mY}-e^m)\\-e^{(m_1+m_2)Y}(1-e^m)\end{Bmatrix}$$
$$+\frac{B\left(C_2-2a_1C_1-b_3C_2(m_1-m_2)^2\right)}{(m_1-m_2)(m_1-m_2-m)(e^m-1)}\begin{Bmatrix}e^{m_1-m_2}(1-e^{mY})+(e^{mY}-e^m)\\-e^{(m_1-m_2)Y}(1-e^m)\end{Bmatrix}$$
$$-\frac{a_1A^2\left((m_1+m_2)^2-S\right)}{2(m_1+m_2)[(2(m_1+m_2)-m](e^m-1)}\begin{Bmatrix}e^{2(m_1+m_2)}(1-e^{mY})+(e^{mY}-e^m)\\-e^{2(m_1+m_2)Y}(1-e^m)\end{Bmatrix}$$
$$-\frac{a_1B^2\left((m_1-m_2)^2-S\right)}{2(m_1-m_2)[(2(m_1-m_2)-m](e^m-1)}\begin{Bmatrix}e^{2(m_1-m_2)}(1-e^{mY})+(e^{mY}-e^m)\\-e^{2(m_1-m_2)Y}(1-e^m)\end{Bmatrix}$$

The Shear Stress :

$$\frac{dU}{dY}=\frac{C_1}{S\sinh m_2}\begin{Bmatrix}e^{m_1(Y-1)}[m_2\cosh m_2Y+m_1\sinh m_2Y]-e^{m_1Y}[m_2\cosh m_2(Y-1)\\+m_1\sinh m_2(Y-1)]\end{Bmatrix}$$

The Shear Stress on the Lower plate :

$$\frac{dU}{dY}|_{(Y=0)}=\frac{C_1}{S\sinh m_2}\left\{m_2(e^{-m_1}-\cosh m_2)-m_1\sinh m_2\right\}$$

The Shear Stress on the upper plate :

$$\frac{dU}{dY}|_{(Y=1)}=\frac{C_1}{S\sinh m_2}\left\{m_1\sinh m_2-m_2(e^{m_1}-\cosh m_2)\right\}$$

The Nussult number (Heat transfer coefficient) :

$$\frac{dT}{dY}=p+me^{mY}\begin{Bmatrix}\frac{p-1}{1-e^m}-p_1(1-e^{2m_1})+p_2(1-e^{m_1+m_2})+p_3(1-e^{m_1-m_2})\\-p_4(1-e^{2(m_1+m_2)})-p_5(1-e^{2(m_1-m_2)}\end{Bmatrix}$$
$$+(1-e^m)\begin{Bmatrix}2p_1m_1e^{2m_1Y}-(m_1+m_2)e^{(m_1+m_2)Y}[p_2-2p_4e^{(m_1+m_2)Y}]\\-(m_1-m_2)e^{(m_1-m_2)Y}[p_3-2p_5e^{(m_1-m_2)Y}]\end{Bmatrix}$$

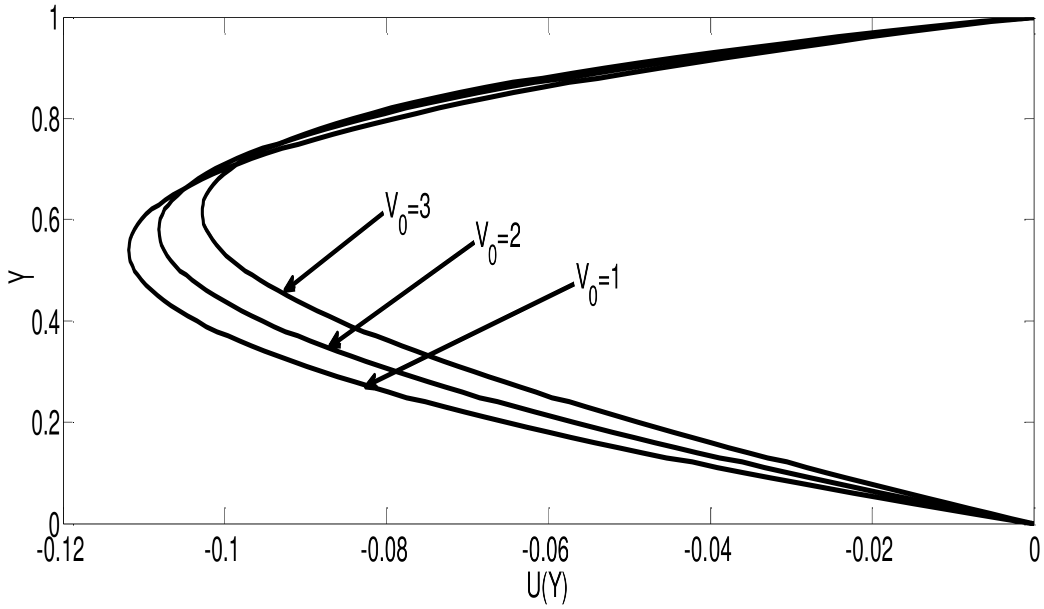
The Nussult number (Heat transfer coefficient) on the lower plate :

$$\frac{dT}{dY}|_{(Y=0)}=p+m\begin{Bmatrix}\frac{p-1}{1-e^m}-p_1(1-e^{2m_1})+p_2(1-e^{m_1+m_2})+p_3(1-e^{m_1-m_2})\\-p_4(1-e^{2(m_1+m_2)})-p_5(1-e^{2(m_1-m_2)}\end{Bmatrix}$$
$$+(1-e^m)\{2p_1m_1-(m_1+m_2)[p_2-2p_4]-(m_1-m_2)[p_3-2p_5]\}$$

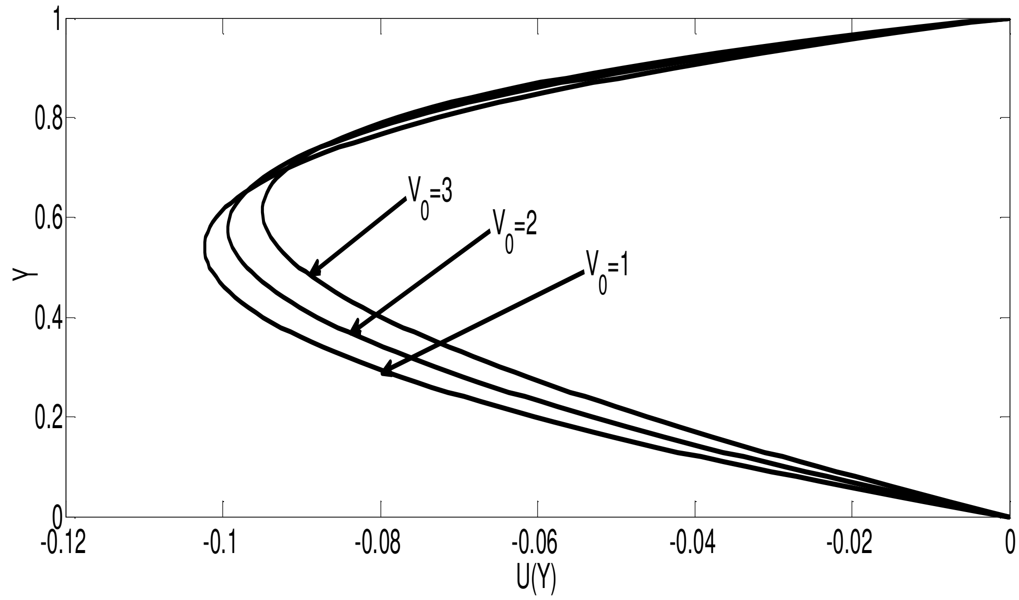
The Nussult number (Heat transfer coefficient) on the upper plate :

$$\frac{dT}{dY}|_{(Y=1)}=p+me^m\begin{Bmatrix}\frac{p-1}{1-e^m}-p_1(1-e^{2m_1})+p_2(1-e^{m_1+m_2})+p_3(1-e^{m_1-m_2})\\-p_4(1-e^{2(m_1+m_2)})-p_5(1-e^{2(m_1-m_2)}\end{Bmatrix}$$
$$+(1-e^m)\begin{Bmatrix}2p_1m_1e^{2m_1}-(m_1+m_2)e^{(m_1+m_2)}[p_2-2p_4e^{(m_1+m_2)}]\\-(m_1-m_2)e^{(m_1-m_2)}[p_3-2p_5e^{(m_1-m_2)}]\end{Bmatrix}$$

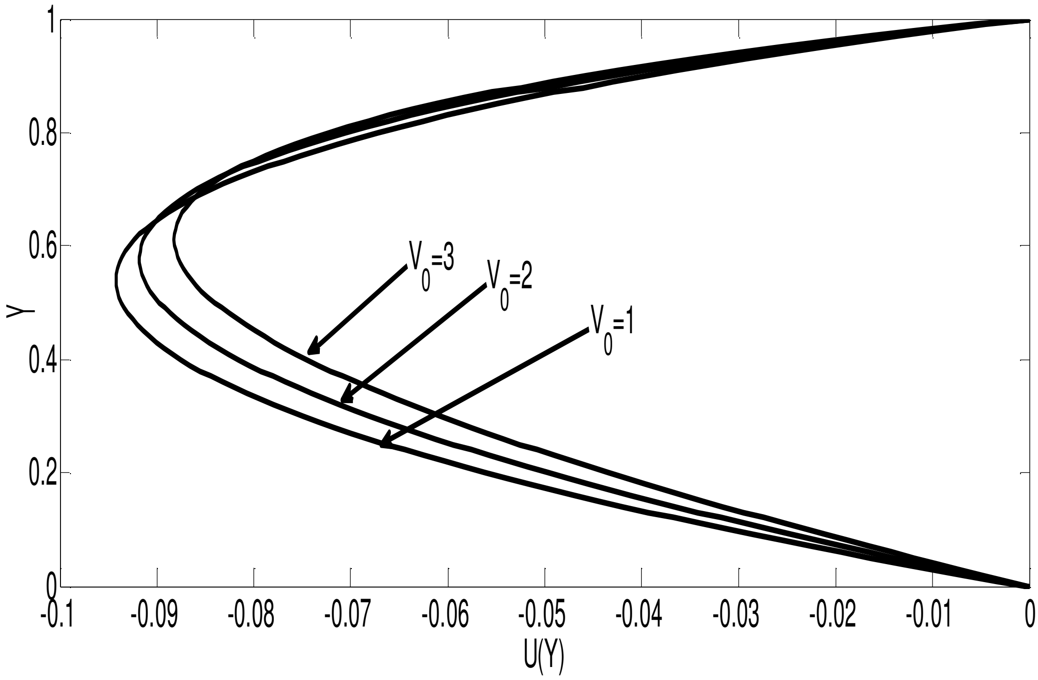
IV. ILLUSTRATIONS



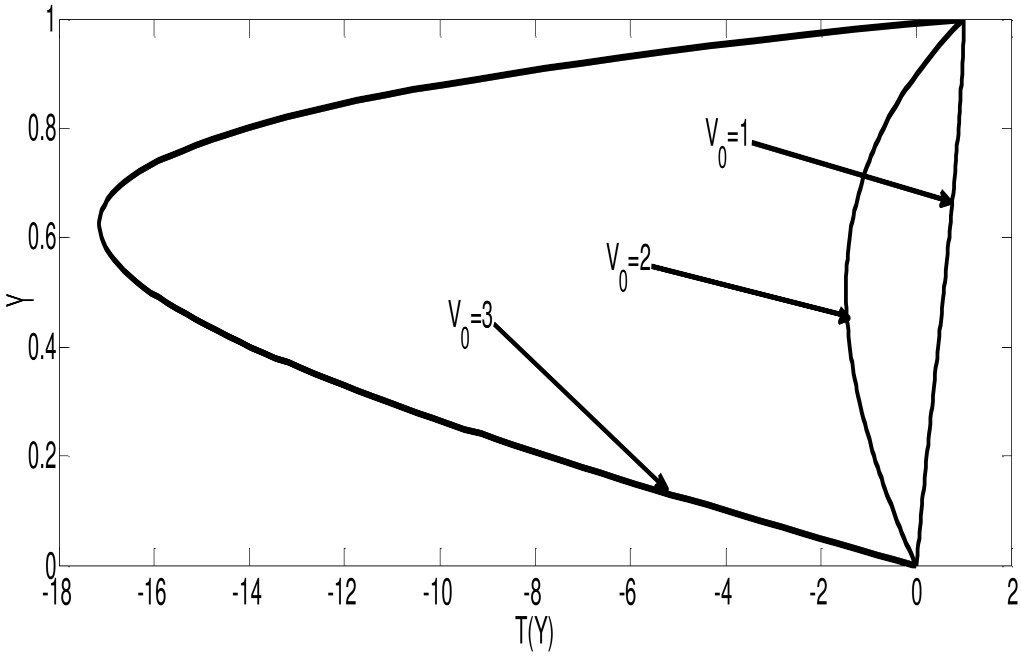
"Figure 1. Variations of the velocity profiles $U(Y)$ with the Darcy's porosity parameter(S=1) and Suction/injection parameter(V_0)"



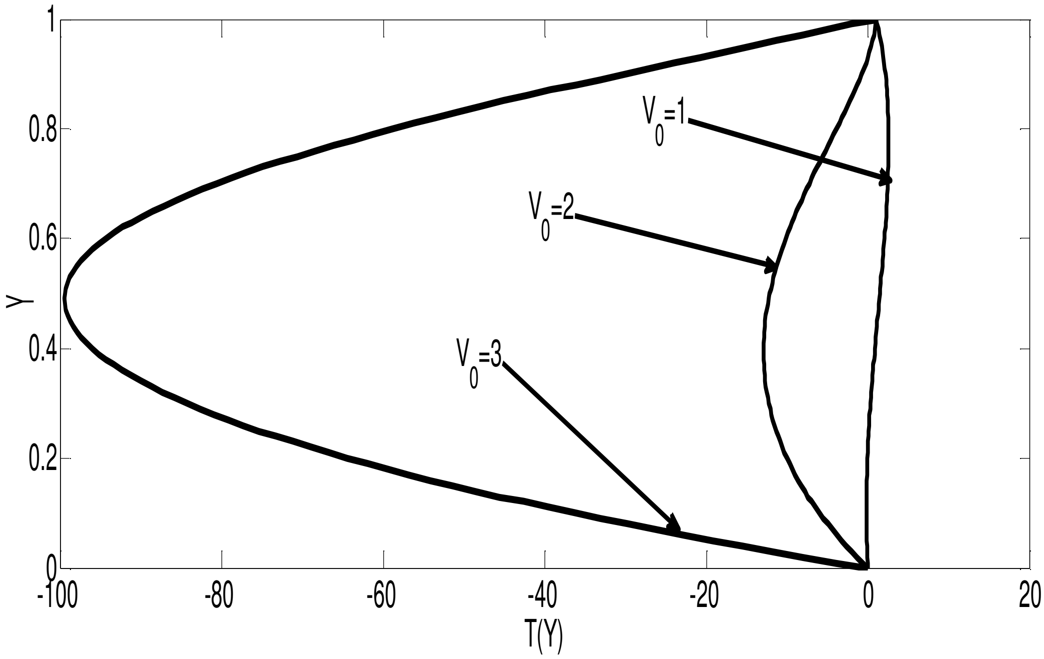
"Figure 2. Variations of the velocity profiles $U(Y)$ with the Darcy's porosity parameter(S=2) and Suction/injection parameter(V_0)"



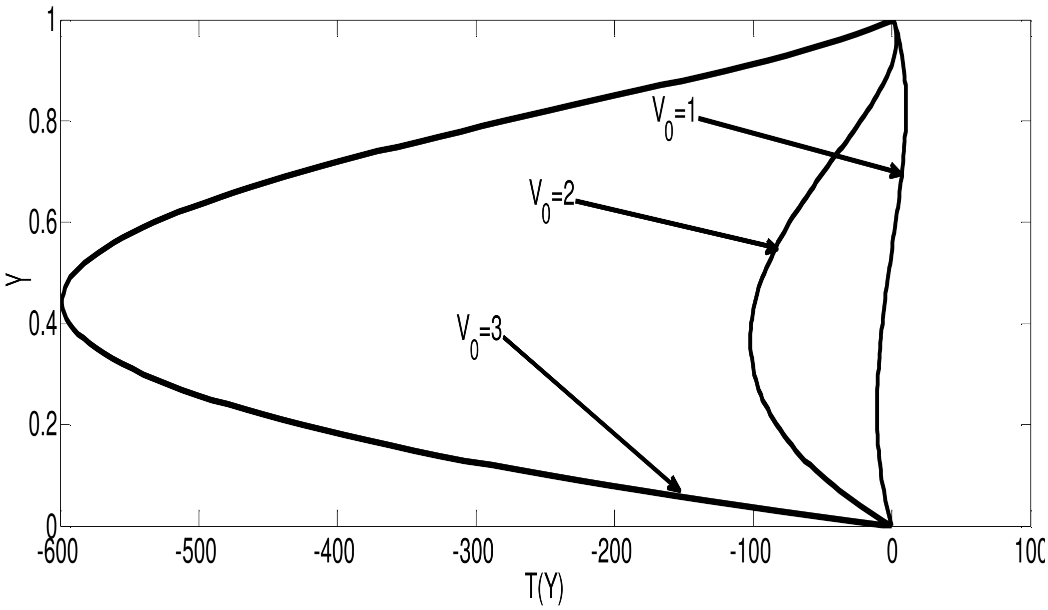
"Figure 3. Variations of the velocity profiles $U(Y)$ with the Darcy's porosity parameter(S=3) and Suction/injection parameter(V_0)"



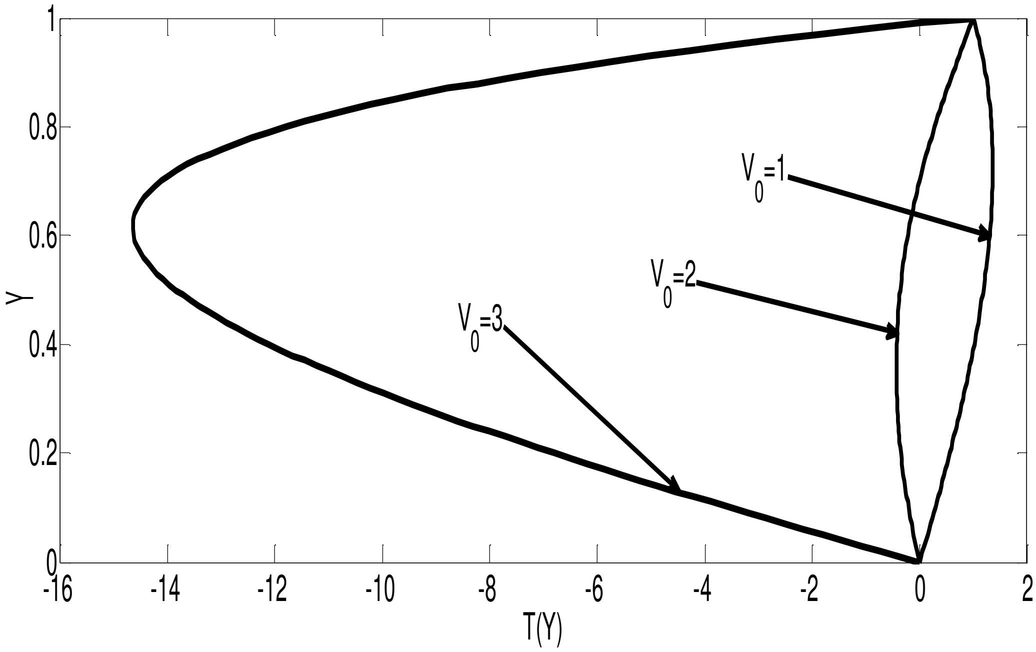
"Figure 4. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3=1), Darcy's porosity parameter(S=1) and Suction/injection parameter(V_0)"



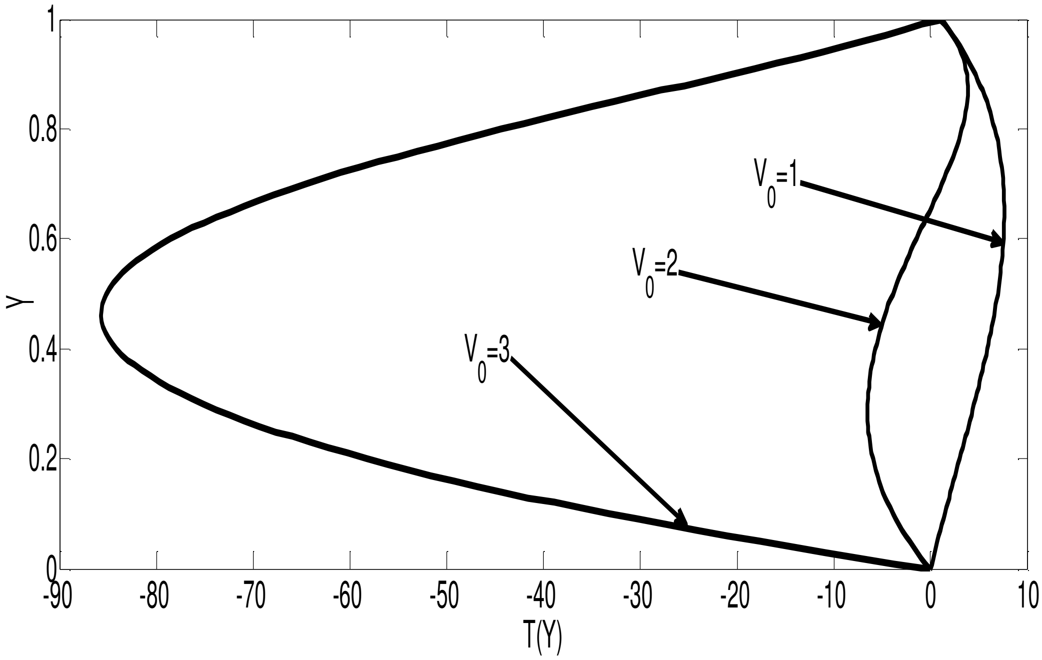
"Figure 5. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3 =1),Darcy's porosity parameter(S=2) and Suction/injection parameter(V_0)"



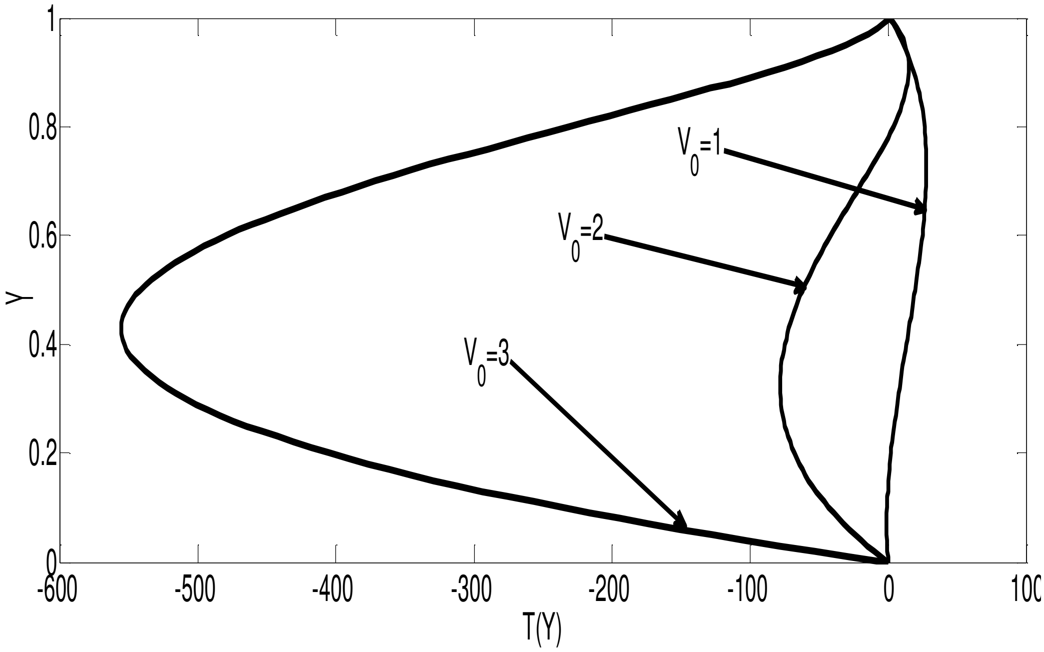
"Figure 6. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3=1), Darcy's porosity parameter(S=3) and Suction/injection parameter(V_0)"



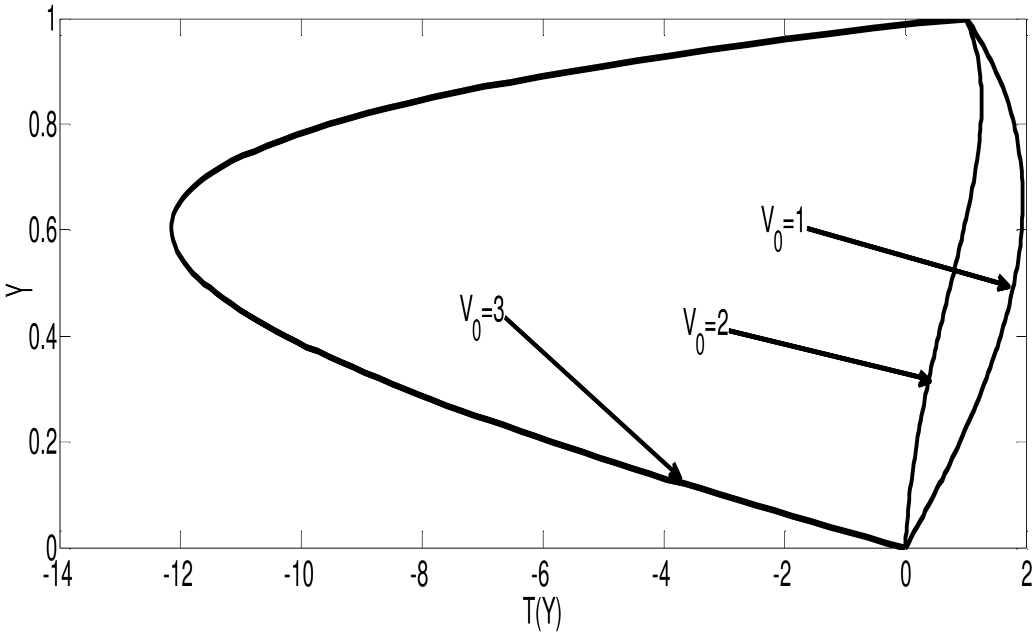
"Figure 7. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3 =3),Darcy's porosity parameter(S=1) and Suction/injection parameter(V_0)"



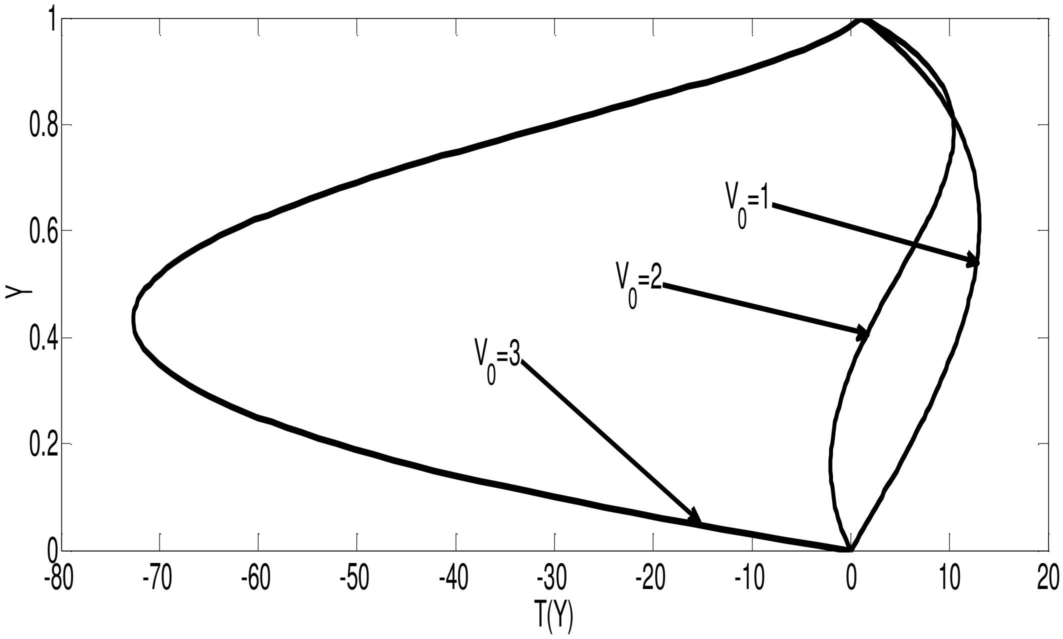
"Figure 8. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3 =3),Darcy's porosity parameter(S=2) and Suction/injection parameter(V_0)"



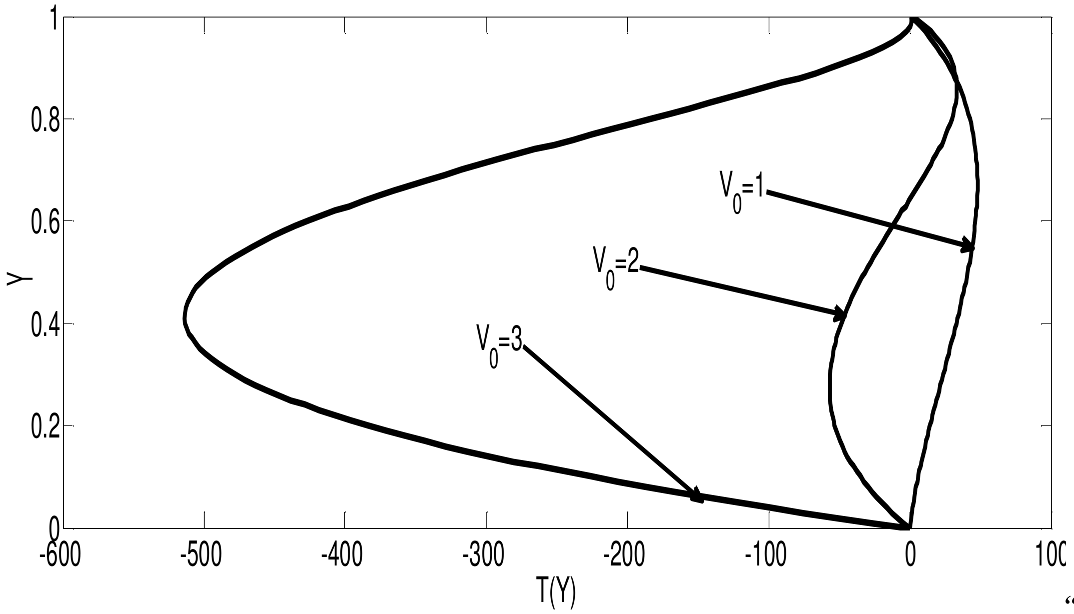
"Figure 9. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3 =3),Darcy's porosity parameter(S=3) and Suction/injection parameter(V_0)"



"Figure 10. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3 =5),Darcy's porosity parameter (S=1) and Suction/injection parameter(V_0)"



"Fig.(11): Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3=5),Darcy's porosity parameter(S=2) and Suction/injection parameter(V_0)"



"Figure 12. Variations of the temperature profiles $T(Y)$ with the Strain thermal conductivity coefficient (b_3=5),Darcy's porosity parameter(S=3) and Suction/injection parameter(V_0)"

V. RESULTS AND DISCUSSION

The effects of various material parameters like Strain thermal conductivity coefficient(b_3), Darcy's porosity parameter(S) and Suction/injection parameter(V_0) on velocity and temperature distributions have been discussed for the fixed values of C_1=1.5, $C_2 = 1$, $p_r = 1$ and $a_1 = 1$.

From the Fig.(1), Fig.(2) and Fig.(3), it is observed that, as the value of Suction/injection parameter(V_0) increases from 1-3, the velocity of the fluid raises from the lower plate up to the middle of the channel, then it is slowly approaches with the velocity of the upper plate. As the velocity approaches to the upper plate, the rate of increase of the velocity up to the middle of the plates is faster than the rate of decrease of the velocity of the fluid near the upper plate. It is also noticed that, as the value of Darcy's porosity parameter(S) increases, the velocity of the fluid slowly moves towards the origin.

It is noticed from the Fig.(4) to Fig.(12) that , the temperature increases, as the value of Suction/injection parameter(V_0) and Strain thermal conductivity coefficient(b_3) both increases. As the value of Darcy's porosity parameter(S) increases and for V_0=1,2, the temperature of the fluid cools down very fast while for $V_0 = 3$, the rate of increase of temperature of the fluid is very slow.

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