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Design of Linear Phase Variable Digital Filter based on Spectral Parameter Approximation and Improved Coefficient Decimation Method

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Abstract—Software defined radios (SDRs) and cognitive radios (CRs) empower mobile communication handsets to sup- port multiple wireless communication standards and services, and improve the spectrum utilization efficiency. SDRs and CRs need multi- standard wireless communication receivers (MWCRs) to integrate existing as well as imminent commu- nication standards into a single generic hardware platform. The limited reconfigurability of the analog front-end, sampling rate constraints of the currently available ADCs and extensive disparities between communication standards' specifications, lead to the shifting of stringent channel(s) selection task to the digital front-end (DFE). Thus, the DFE needs either variable digital filters (VDFs) that provide variable lowpass (LP), high- pass (HP), bandpass (BP) and bandstop (BS) responses and/or reconfigurable filter banks that provide independent and indi- vidual control over the bandwidth and the center frequency of subbands.The linear phase VDF is designed by combining new Improved Coefficient Decimation Method(ICDM) with Spectral Parameter Approximation (SPA),termed as SPA-ICDM VDF.In the proposed filter bank, subbands of desired bandwidths are obtained by the spectral subtraction of the lowpass and high- pass frequency responses obtained after performing coefficient decimation operations on the prototype filter, using appropriate decimation factors.

Keywords-Improved Coefficient Decimation Method(ICDM); spectral parameter approximation (SPA); variable digital filters (VDFs)

I. INTRODUCTION

Variable digital filters (VDF) are filters designed such that some spectral characteristic of the filter can be adjusted by changing one or more tuning parameters. VDFs are used in many areas of signal processing and communica- tions. Examples are variable fractional-delay filters used in digital receivers and variable cutoff-frequency filters used extensively in audio production tools. More recently, much research has gone into the design of FIR VDFs, desirable for guaranteed stability, perfect linear phase, and the increasing accessibility of optimization routines to aid in coefficient calculation.

The main goals in VDF design are high tuning accu- racy, large tuning range, low order, and efficient coefficient calculation (fast tuning time).Currently there are techniques such as the spectral transformation technique that mostly have low order and have fast tuning times, but they have low tuning accuracy. Thus most research has gone into spectral parameter approximation, a technique that has fast tuning time and high accuracy for arbitrary parameters, however, the order is very high.[17-19] VDFs are useful in appli- cations such as channelization in software-defined radios, spectrum sensing in emerging cognitive radios, adaptive systems, biomedical applications, and reconfigurable

filter bank design. For such applications, the VDF must be able to produce tunable lowpass (LP), highpass (HP), bandpass (BP), and bandstop (BS) responses without hardware reim- plementation or coefficient update. A number of linear and nonlinear-phase VDF designs are available. The allpass transformation (APT)-based VDFs (APT-VDFs) [20-25] are obtained by replacing each unit delay of a digital filter by an APT structure of an appropriate order, and they allow on- the-fly control over the cutoff frequency. A low-complexity APT-VDF in provides variable LP, HP, BP, and BS responses from a fixed-coefficient prototype filter. However, because of nonlinear-phase characteristics, APT-VDFs are not preferred for many signal processing and wireless communication ap- plications. A frequency-transformation-based linear-phase VDF is proposed, and it is further extended.[15-19] The modified-frequency- transformation-based VDF (MFT-VDF) provides variable LP, HP, BP, and BS responses from a fixed- coefficient prototype filter but only over a limited section of the Nyquist band. The VDFs offer a wide cutoff frequency range with sharp transition bandwidth (TBW), but the group delay is huge.
Spectral-parameter-approximation-based VDFs (SPA- VDFs) are designed using the Farrow structure. Their advantages over other VDFs include fixed TBW, lower group delay, fewer variable multiplications (VMs), and high accuracy. SPA- MCDM-VDF provides variable LP, HP, BP, and BS responses with unabridged bandwidth control over the entire Nyquist band without hardware reimplementation or coefficient update. A low-complexity VDF designed by deft integration of the SPA-VDF with the improved coef- ficient decimation method (ICDM) is proposed. It will be referred to as SPA-ICDM-VDF.

This paper is organized as follows. Section II gives a brief review of SPA-MCDM-VDF. Section III describes design example and discussions in Section IV followed by Sections V which concludes the work.

A. SPA-VDF

The block diagram of the SPA-VDF is shown in figure where $H_i(z)$, $0 \leq i \leq L$, are sub-filters and α is the control- ling parameter which controls either the cut-off frequency or the fractional delay of the SPA-VDF. This approach is known as spectral parameter approximation since the VDF is a weighted combination of fixed-coefficient FIR sub-filters and the weights are directly proportional to the spectral parameter.[12] The SPA technique is initially proposed to design VDFs with tunable fractional delays and they

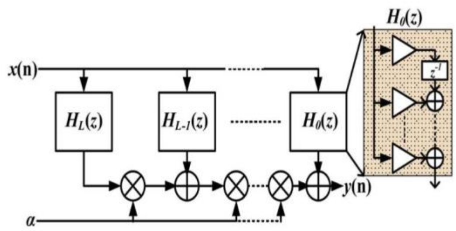

Figure 1. SPA-VDF with FIR sub filters in transposed direct form

employed Farrow structure which provides online tuning for the phase delay of the input signal. Here, first (L+1) sub- filters $H_i(z)$, $0 \leq i \leq L$, are designed offline and optimized for a given range of delay. Then, their impulse responses are interpolated by L^{th} order polynomial using α as the variable. This technique is then extended to the design of the VDF with tunable frequency specifications such as cut- off frequency. In case of the SPA-VDF with tunable cut- off frequency, $H_i(z)$, $0 \leq i \leq L$, are fixed- coefficient sub-filters with distinct cut-off frequencies and α is the controlling parameter which decides the cut-off frequency of the SPA VDF.
Different number of approaches has been proposed to determine filter coefficients, $h_k(n)$, so

that the frequency response of H(z, α) will approximate the desired response as a function of α. In the first approach, the least squares or Parks-McClellan filter is designed for each tuning points and then polynomial curve fitting is done to obtain hk(n) where $0 \le k \le L$ and $0 \le n \le N$. Subsequently, many optimization techniques such as minimax approximation, linear programming, least square, weighted least square and constrained least square are proposed. The SPA-VDFs has advantages such as fixed TBW, lower overall group delay, few adjustable parameters resulting in a simple updating routine, fast tuning time and high accuracy compared to frequency transformation VDFs.

It can be observed that additional output responses can be easily obtained from a fixed-coefficient prototype filter. Each extra output requires only L extra multiplications which are significantly smaller than the transformation based VDFs. However, the complexity of the SPA-VDF is very high, almost 8-10 times that of frequency transformation based VDFs .

Also, coefficient values of sub-filters increase exponentially with their order which may impose constraints when fixed-point implementation is needed.

Since the order of sub-filters depends on the cut-off frequency range, SPA- VDFs are preferred for application which requires narrower cut-off frequency range. Most of the current research on SPA-VDFs is focused on algorithms for optimizing sub-filter coefficients thereby improving the mean square error and reducing offline processing time.

B.CDM, MCDM & ICDM

1) CDM: Coefficient decimation method (CDM) is used to obtain reconfigurable and low complexity FIR filters. Two coefficient decimation operations, one to vary the pass band width of the prototype filter (termed as CDM- II) and another to generate multi-band frequency responses (termed as CDM-I) are used. A multi-stage [5] coefficient decimation based FB (MS-CDFB) employs CDM-I, CDM-II and frequency response masking filters for obtaining uniform FBs with reconfigurable subband BWs. In [6], a low power, reconfigurable filter architecture can be used for non-uniform channelization. It is based on CDM-II, interpolation and frequency response masking and eliminates the use of CDMI operation which is required in MS-CDFB. In the conventional CDM, the coefficients of a lowpass FIR filter (termed as model filter) are decimated by M, i.e., every M^{th} coefficient is retained and the others replaced by zeros, which results in an FIR filter with a multiband frequency response. The frequency response of the resulting filter has bands with center frequencies at 2πk/M, where k is an integer ranging from 0 to M-1. If $H(e^{j\omega})$ denotes the Fourier transform of the modal filter coefficients, then the Fourier transform of the modified coefficients is given by,

$$H'(e^{j\omega})=1/M\sum_{k=0}^{M-1} H(e^{j(\omega-\frac{2\pi k}{M})}) \qquad (1)$$

From above equation, we can see that the frequency response of the modified coefficients is scaled by M and the original frequency spectrum is replicated at the locations 2πk/M, where k = 0 to M-1. This operation is termed as CDM-I and the mathematical derivation for this operation is given in literature review. After performing the CDM-I operation by a decimation factor M, if all the retained coefficients are grouped together by discarding the zero coefficients in between, a frequency response similar to the modal filter response is obtained with the passband and transition band widths M times that of the modal filter. This operation is called as CDM-II.

2)MCDM: In MCDM using decimation factor M, every Mth coefficient of the prototype filter, $H(e^{j\omega})$ is retained, and others are replaced by zeros. Then, the sign of every alternate retained coefficient is reversed which is given by [14].As a result of this operation, an FIR filter with a multi-band frequency response is obtained with center frequencies at (2k+1) π/M, where k is an integer ranging from 0 to M-

1[14]. If H($e^{j\omega}$) denotes the Fourier transform of the modal filter coefficients, then the Fourier transform of the modified coefficients is given by,

$$H'(e^{j\omega})=1/M\sum_{k=0}^{M-1} H(e^{j(\omega-\frac{(2k+1)\pi}{M})}) \qquad (2)$$

Thus, the frequency response of the modified coefficients is scaled by M and the original frequency spectrum is replicated at the locations (2k+1π/M, where k = 0 to M- 1. The mathematical derivation of this operation is given in literature review. We name this operation as modified coefficient decimation method I (MCDM-I).

After performing MCDM-I operation by a decimation factor M, if the retained coefficients are grouped together by discarding the zero coefficients in between, a highpass filter is obtained with its passband and transition band widths M times that of the modal filter. We name this operation as modified coefficient decimation method II (MCDM-II). If the modal filter and the decimation factor value are the same, the corresponding CDM-II and MCDM-II operations are mathematically related as

$$h''(n)=(-1)^n h'(n) \qquad (3)$$

where h'(n) represents the coefficients of the filter obtained after CDM-II operation, h''(n) represents the coefficients of the filter obtained after MCDM-II operation and N is the filter order. From (6), it can be noted that the high pass filter obtained after MCDM-II is the inverse of the low-pass filter obtained after CDM-II. From (4), (5) and (6), we can also note that for M=1, CDM-II gives the modal filter as the output whereas MCDM-II gives inverse of the modal filter as the output.

3) ICDM: quad The combination of CDM-I and MCDM- I operations are termed as improved coefficient decimation method I (ICDM-I), and the combination of CDM-II and MCDM-II operations as improved coefficient decimation method II (ICDM-II) respectively.

$$\text{ICDM-I} = \text{CDM-I} + \text{MCDM-I} \qquad (4)$$

$$\text{ICDM-II} = \text{CDM-II} + \text{MCDM-II}. \qquad (5)$$

From the multiband frequency responses obtained after ICDM-I operations, individual subbands with identical BWs can be isolated by the use of frequency response masking filters and spectral subtraction [4, 9].

However, the use of masking filters can add to the complexity involved in implementing these methods. We obtain lowpass and highpass frequency responses with varying passband widths after performing ICDM-II operations without employing masking filters. Frequency subbands with identical as well as non-identical BWs can be obtained from these low- pass and highpass output frequency responses by spectral subtraction/addition.[6-11].

II. SPA-MCDM VDF

The goal of the proposed SPA-MCDM-VDF is to provide tunable LP, HP, BP, and BS responses over the entire Nyquist band without the need for time-consuming hardware reimplementation or coefficient update. This is achieved by deftly integrating LP SPA-VDF with the MCDM. The SPA- MCDM- VDF is not just a straightforward integration of these two techniques.[1-5] In fact, the SPA-MCDM-VDF is carefully designed by exploiting the architectural advantages of the Farrow structure as well as the exclusive multiband response capability of MCDM. The design of the SPA- MCDM-VDF is explained in detail below.

A. Basic Principle:

Consider the four equal parts of the Nyquist band: (0-0.25π), (0.25π-0.5π),(0.5π-0.75π), and (0.75π-π). The basic principle of SPA-MCDM-VDF is that using an LP prototype filter with cutoff frequency, ωc , three additional LP responses with cutoff frequencies (π - ωc), (0.5π - (π - ωc), (0.5π + ωc) and (0.5π - ωc)c) and (0.5π + ωc) can be obtained via MCDM. Based on this principle, the prototype filter in the MCDM is replaced with the prototype SPA- VDF, Hα(z), that provides variable LP responses with TBW of TBW_d and unabridged control over cut off frequency ω_{cpa} ,in the second quarter,i.e.,$0.25\pi \leq \omega_{cpa} \leq 0.5\pi$,Then, LP responses with ωc in each quarter of the Nyquist band, i.e.,$(TBW_d/2)\pi \leq \omega_c \leq [1-(TBW_d/2)]\pi$ are obtained as follows:

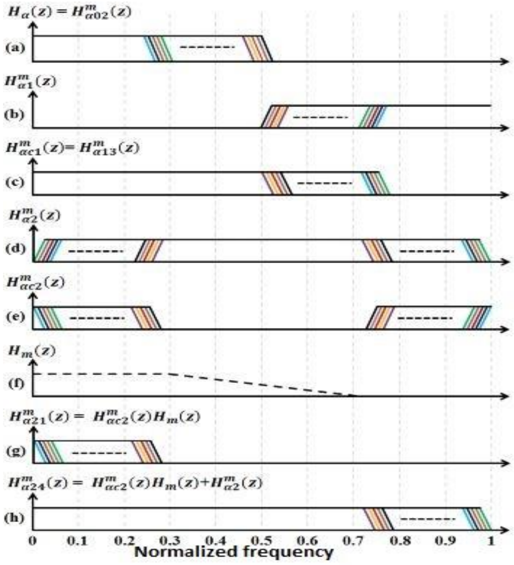

Figure 2. Frequency responce of LP responces over entire bandwidth

1) LP Response in the Second Quarter: Hα(z) provides LP responses in the second quarter. The frequency response of Hα(z) is shown in Fig. 2(a). It is also denoted by $H\alpha02^m(e^{j\omega c})$ where subscripts '0' and '2' represent D=0 and the second quarter, respectively.

2) LP Response in the Third Quarter: Using the MCDM with D = 1, the HP responses, $H\alpha1^m$ (z) with cutoff frequencies in the third quarter can be obtained as shown in Fig. 2(b). By complementing $H\alpha1^m$ (z),we have LP responses $H\alpha13^m(e^{j\omega c})$with $\omega_c = \pi-\omega_{cpa}$ .Mathematically these operations can be expressed as;

$$H\alpha13^m(e^{j\omega c})= e^{-j\omega cpa(N-1/2)}-H\alpha(e^{j\omega cpa-\pi}) \quad (6)$$

3) LP Response in the First Quarter: Using MCDM with D = 2, we have BP response, $H\alpha2^m$ (z) amd its complementary BS response $H\alpha c2^m$(z) as shown in figure. The higher frequency subband of $H\alpha c2^m$(z) can be masked using the N_m^{th} order masking filter,H_m (z),shown in fig..This results in LP responses,$H\alpha21^m(e^{j\omega c})$ with $\omega_c=(0.5\pi-\omega_{cpa})$ i,e, $(TBW_d/2)\pi \leq \omega_c \leq 0.25\pi$ as shown in figure. Mathematically,

$$H\alpha21^m(e^{j\omega c})= H_2^m(e^{j\omega c})H_m(e^{j0.5\pi}) \quad (7)$$

4) LP Response in the Fourth Quarter: Adding the responce $H\alpha2^m$(z) to $H\alpha21^m$(z),the LP responses with $\omega_c = (0.5\pi + \omega_{cpa})$ i.e,$0.75\pi \leq \omega_c \leq [1-(TBW_d/2)]\pi$.

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Mathematically,

$$H\alpha 24^{m}(e^{j\omega c}) = H\alpha 21^{m}(e^{j\omega c}) + H\alpha 2^{m}(e^{j\omega c})e^{-j\omega cpa(Nm-1/2)} \quad (8)$$

The HP responses can be obtained by complementing corresponding LP responses. Furthermore, the BP responses with lower and upper cutoff frequencies of ωl and ωh , respectively, can be obtained by subtracting the LP response with ωl from another LP response with ωh.Likewise BS responces can be obtained.The TBW of all the response is fixed and equal to TBW_d .Since the range of ωc of the LP response is $(TBW_d/2)\pi \leq \omega_c \leq [1-(TBW_d/2)]\pi$,which spans the entire Nyquist band,ωl and ωh of BP and BSre- sponces are not restricted to limited values unlike other VDFs.

III.DESIGN EXAMPLE

Consider the first design example with TBW_d =0.2π,δ_{sd}=-50dB and δ_{pd}=0.1dB. Then range of ω_c is 0.1π to 0.9π.For these specifications, the SPA-MCDM-VDF is designed with L = 5, N = 32, Nm = 18, and the corresponding variable LP responses are shown in above figure, where responses in blue, green, red, and yellow are the those obtained using the respective steps 1-4 in Section III.

IV. RESULTS AND DISCUSSION

SPA-MCDM-VDF provides tunable lowpass, high- pass,bandpass, and bandstop responses over the entire Nyquist band.Here lowpass filter is implemented in all quarters of the nyquist band.The bandpass responses with cutoff frequencies corresponding to all quarters also obtained by performing modified coefficient decimation method.

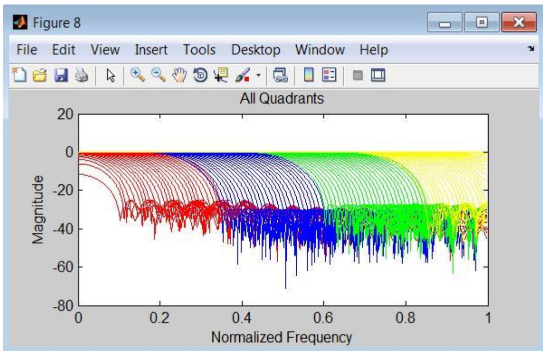

Figure 3. Variable LP responses using SPA-MCDM-VDF

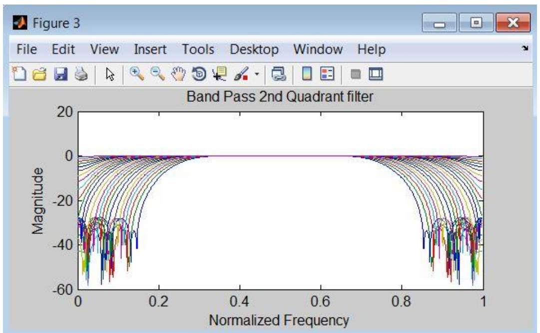

Figure 4. BP responses in the 1st and 4th quarter using SPA-MCDM

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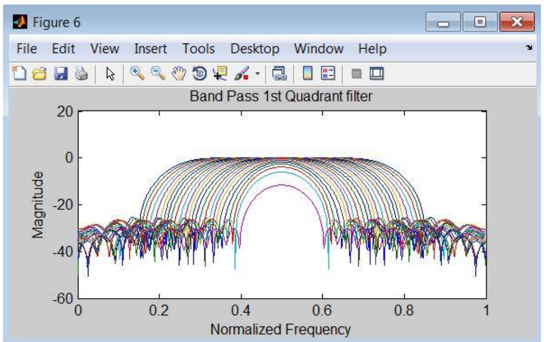

Figure 5. BP responses in the 2nd and 3rd quarter using SPA-MCDM

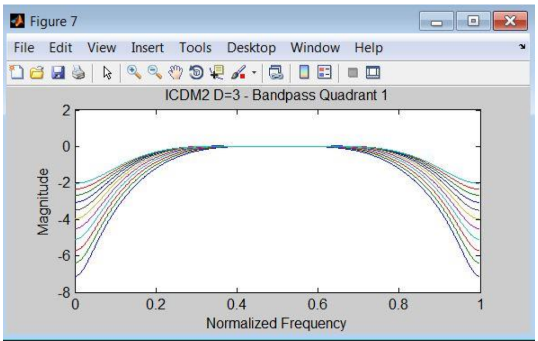

Figure 6. BP responses in the 1st and 4th quarter using SPA-ICDM

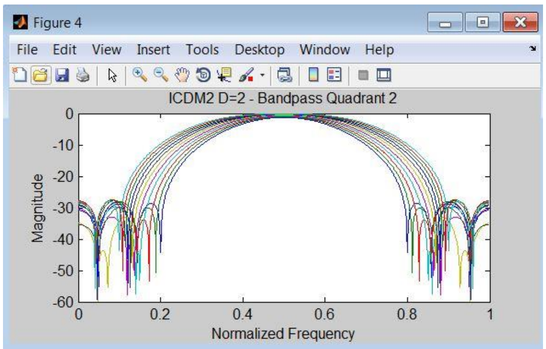

Figure 7. BP responses in the 1st and 4th quarter using SPA-ICDM

Method Used	Decimation Factor, D=2		Decimation Factor, D=3	
	TB=0.2π	TB=0.08π	TB=0.2π	TB=0.08π
SPA-MCDM-VDF	9.058	80	9.058	80
SPA-ICDM-VDF	8.000	20	5.34	13

Figure 8. Comparison of Group Delays

complexity involved in implementing these methods. We can obtain lowpass and highpass frequency responses with vary- ing passband widths after performing ICDM-II operations without employing masking filters. Frequency subbands with identical as well as non-identical BWs can be obtained from these lowpass and highpass output frequency responses by spectral subtraction/addition,thus we get SPA-ICDM- VDF.Thus SPA-ICDM-VDF provides bandpass response which have the advantage of lower group delay and lower implementation complexity.

V. CONCLUSION

The design examples demonstrated that SPA-MCDM- VDF provides variable LP, HP, BP, and BS responses with unabridged and independent control over the cutoff frequencies on the entire Nyquist band.

Since the use of masking filters can add to the complexity involved in implementing these methods, SPA-ICDM-VDF is the best method with offers lower group delay along with lower complexity hard-ware implementation.

In this brief, a low-complexity linear- phase VDF by integrating SPA-VDF with the ICDM has been presented, and it is termed as SPA-ICDM-VDF.

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