Nonlinear Identification of A Wireless Control System: Comparison of NARX Model Results

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Abstract—This work has been carried out to comparison of two types of nonlinear models (wavelet network NARX and sigmoid network NARX) were applied for a process simulator that was used for the wireless control. Wireless input/output data obtained from the Cussons P3005 type process control simulator which three different temperatures (T2, T3 and T4) were selected as the controlled variables and the heater capacity was chosen as the manipulated variable. Wireless temperature experiments were achieved by using MATLAB/Simulink program and wireless data transfer during the experiments were carried out with radio waves at a frequency of 2.4 GHz. The two nonlinear models were developed with the aid of System Identification Toolbox of MATLAB using the data acquired from a unit step change on the heater capacity of the process simulator. The model orders used for the estimation of the model coefficients were determined with the aid MATLAB. According to the higher fit value and lower loss function observed in the case of the NARX model developed using wavelet network model has been found to be better than sigmoid network model. In addition, based on the simulation results, these two NARX models getting to the steady-state without any oscillations but wavelet network NARX models were discovered to be better than sigmoid network NARX models due to fastest rise time and fastest response time for this wireless control system.

Keywords—System identification, MATLAB/Simulink, wavelet network NARX model, sigmoid network NARX model, wireless process control

I. INTRODUCTION

Before the design of a controller, it is necessary to achieve system identification of process. System identification is concerned with the development of regression models that describe the dynamics of a system from measurements of its inputs and outputs [1]. In general, the process of system identification can be decomposed into two steps: the selection of an identification model and an estimation of the model’s parameters, of which, the parameter estimation plays a relatively more important role since a specific class of models that can best describe the real system can usually be derived by mechanism analysis of industrial processes [2]. Identification of nonlinear systems is significant importance to the scientific community as most of the real systems and chemical processes offer a nonlinear behavior [3]. The identification and control of nonlinear systems have been widely studied in recent years. The nonlinear modeling and identification of real systems, including, small-scale unmanned helicopter [4], energy systems; proton exchange membrane fuel cell [5] and wind turbines [6], chemical processes [7], reactive distillation column [8], mechanical systems; automotive dampers [9], tractor–trailer system [10].

Constructing such a NARX model involves three steps [11]; (1) model input selection. More specifically, the unknown lags need to be determined. Statistical tests and regression methods are among the popular approaches [12]; (2) choice of mapping function f (y(t)). Polynomials, radial basis function networks, neuro fuzzy networks and wavelet networks are popular options. Though some suggestions are made on the function selection [13], no unified framework is available; (3) parameter identification in function f (y(t)). This requires the specific expression of the model (1).

In this study, comparison of two nonlinear models for wireless control system using wavelet network model and sigmoid network model as the nonlinearity estimators with the aid of System
Identification Toolbox of MATLAB (MathWorks 2011). Wireless temperature experiments were achieved by using MATLAB/Simulink program and wireless data transfer during the experiments were carried out using radio waves at a frequency of 2.4 GHz. Two nonlinear model performances are compared with the calculated fit values, loss function values and simulation results of three different temperature points on this wireless control system.

II. EXPERIMENTAL SYSTEM AND PROCEDURE

1) **Description of Wireless Data Generation System**

The process control simulator consists of two main units, an instrument console and a framework carrying the process equipment which is shown in Figure 1. The instrument console contains the electronic flow, level, temperature controllers and electrical switchgear. It is connected to the process equipment by several cable assemblies. The process equipment consists of a water tank, water circulating pump, electrical water heater, two vessels, two electrically positioned control valves and a heat exchanger. In process control simulator, twelve manual valves are available for different process experiment loops. In the process simulator, temperature measurement and control can be made at four different points which are first tank (T1), heater output (T2), second tank input (T3) and second tank output (T4).

The wireless system developed for transferring data between the computer and the control panel. To achieve the data transfer between computer in Process Control Laboratory and the process simulator in Unit Operations Laboratory, by using the two antennas are found in the laboratory connected to the computer and outside connected to the process simulator. Control valves outputs are connected to the modules, the necessary calibrations are made. The water is pumped via the electrical heater into the reactor up to a certain level. The water then flows back to the sump tank via the cooler. Heat is fed to the water by the heater and residual heat removed by the cooler so as to return the sump tank water temperature to a suitable base level. Heater which is connected on-line to the computer is used as a manipulated variable [14].

Wireless temperature experiments were carried out using MATLAB/Simulink program and wireless data transfer during the experiments were carried out using radio waves at a frequency of 2.4 GHz. MATLAB/Simulink block diagram was used for the on-line wireless temperature experiments with a computer in the office. There have four moduls, wireless on/off block for the wire or wireless experiments, numerical or graphical display blocks of process parameters, blocks of giving numerical values of valve openness, blocks of stored errors on the MATLAB/Simulink block diagram shown in Figure 2.

The wireless data generated by operating the process simulator described above and shown in Figure 1. Wireless data were used for the development of the models of the three temperatures at different points on the process simulator using Process Identification Technique. The two NARX models were determined with the aid of System Identification Toolbox of MATLAB program. Wireless temperature experiments were carried out on process simulator during the 1500s time period. First 300s the heater operated % 10 heating capacity for the temperature is expected to become at steady-state. The fluid flow was obtained by running the pump when the level control valve was % 35 opening. The cooling water was opened after liquid level was fixed value. The heater output temperature was expected to become at steady-state while % heating capacity was on. A step change effect was performed as input signal which apply the heater capacity between % 10-70 values and T2, T3 and T4 temperature changes with time as output signal which selected as the controlled variables while the heater capacity was chosen as the manipulated variable. These effects were given to the heater and output temperatures taken with MATLAB/Simulink during the experiments shown in Figure 2 [14].
Figure 1. Experimental system: Process simulator, control panel and computer on-line connected to the process simulator with wireless technology [14]

Figure 2. MATLAB/Simulink block diagram for wireless temperature experiments [14]
2) Nonlinear System Identification Model

A large class of nonlinear dynamic systems can be described by a nonlinear black-box model structure which is referred to as “Nonlinear AutoRegressive with eXogenous inputs (NARX) model” used in this research for prediction the temperatures of three different points on process simulator is an extension of a linear ARX model. A linear SISO ARX model has the structure that is given as;

\[ T(t) + a_1T(t-1) + a_2T(t-2) + \ldots + a_{na}T(t-na) = b_1R(t) + b_2R(t-1) + \ldots + b_{nb}R(t-nb+1) + e(t) \]  

(1)

where \( na \) is the number of outputs and \( nb \) is the number of inputs. The input delay \( nk \) has been set to zero in the formulation in order to simplify the notation. This structure shown in Eq. (2) implied that, with only the linear ARX model and rewriting the equation as a output temperature;

\[ T_r(t) = [-a_1, -a_2, \ldots, -a_{na}, b_1, b_2, \ldots, b_{nb}]^T[T(t-1), T(t-2), \ldots, T(t-na), R(t), u(t-1), \ldots, R(t-nb-1)]^T \]  

(2)

where \( T(t-1), T(t-2), \ldots, T(t-na), R(t), R(t-1), \ldots, R(t-nb-1) \) are delayed output and input variables, called regressors. The linear ARX model thus predicted the current output \( T_r \) as a weighted sum of the regressors. This structure was further extended to create a nonlinear form by utilizing a more flexible nonlinear mapping function instead of the weighted sum that represented a linear mapping. The form of the nonlinear ARX model is;

\[ T_r(t) = f(T(t-1), T(t-2), T(t-3), \ldots, R(t), R(t-1), R(t-2), \ldots) \]  

(3)

where \( f \) is a nonlinear function. Inputs to \( f \) are the model regressors. The nonlinear identification functions used in this work were wavelet network and sigmoid network. Shown in Figure 3 below is a block diagram representing the structure of the Nonlinear ARX model used for the development of the nonlinear black-box models involving the heater capacity (input) and three temperatures (T2-T3-T4) (outputs).

As can be observed from the Figure 3, the structure of the NARX model consisted of both a linear and a nonlinear function. In the calculation aspect of the nonlinear ARX model, the three temperatures of wireless control system were estimated by first computing the regressors from the current and past input values and past output data using both the linear and the nonlinear function blocks of the nonlinearity estimator. Further, the nonlinearity estimator block mapped the regressors to the model output using a combination of nonlinear and linear functions. The nonlinearity estimator block included linear and nonlinear blocks in parallel as in;

\[ f(x) = LT(x-r) + d + g(Q(x-r)) \]  

(4)
\( x \) is a vector of the regressors. \( LT(x)+d \) is the output of the linear function block and is affine when \( d \neq 0 \). \( d \) is a scalar offset. \( g(Q(x-r)) \) represents the output of the nonlinear function block. \( r \) is the mean of the regressors \( x \). \( Q \) is a projection matrix that makes the calculations well-conditioned. The exact forms of \( f(x) \) were based on wavelet and sigmoid networks that were used in this study. Wavelet network, coded using the \texttt{wavenet} command in \textsc{MATLAB} R2011a, is a class representing wavelet network nonlinearity estimator for \textsc{NARX} models whereas sigmoid network which was coded in the same \textsc{MATLAB} version using the \texttt{sigmoidnet} command has a structure which is given as:

\[
g(x) = \sum_{k=1}^{n} \alpha_k \kappa(\beta_k (x-\gamma_k))
\] (5)

where \( \kappa(x) = (e^x + 1)^{-1} \) is the sigmoid function and \( \beta_k \) is a row vector such that \( \beta_k (x-\gamma_k) \) is a scalar. Estimating a \textsc{NARX} model is the computation of the model parameter values, \( L \), \( r \), \( d \), \( Q \), and other parameters specifying \( g \). The resulting models developed using the \texttt{nlarx} command are \texttt{idnlarx} objects that stored all the model data, including model regressors and parameters of the nonlinearity estimator \cite{11}.

### 3) Determination of Model Orders

The determination of appropriate model orders is very important when developing any black-box model. During the model development, the optimum values of these model orders are necessary to be determined in order to avoid under-fitting or over-fitting of the developed model equation. In this study, the MDL (Rissanen’s Minimum Description Length) criterion, shown in Eq. (6) below, was used for the optimum selections of these model orders because it allows the shortest possible description of the observed data. In Eq. (6) \( V \) is the loss function; \( d \) is the total number of parameters in the structure; and \( N \) is the number of data points used for the estimation.

\[
MDL = V \left( 1 + \frac{d \log(N)}{N} \right)
\] (6)

### III. RESULTS AND DISCUSSION

The wireless data acquired from the experiment carried out in the process simulator are as shown in Figure 4. It can be observed from Figure 3 that a change in the heater capacity between \% 10-70 values to a unit step resulted in a change in the temperatures of the T2, T3 and T4. Using the wireless data acquired from the experiment, the model orders were estimated by optimizing the MDL criterion. The optimum model orders obtained from the optimization of the criterion were \( na=2 \), \( nb=2 \) and \( nk=1 \). Thereafter, the model orders were used to develop the nonlinear models (wavelet network \textsc{NARX} and sigmoid network \textsc{NARX}) for the process simulator in \textsc{MATLAB} environment. The developed two nonlinear models were then simulated and their simulated results were compared to the T2, T3 and T4 temperatures on the process simulator. Simulated temperatures were compared with each other by calculating their fit values (Eq. (7)) and loss function values (Eq. (8)). The calculated fit values and loss function values are as shown in Table 1 and Table 2, respectively:

\[
\text{fit value} \ (\%) = \frac{1 - \text{norm}(Ts - Te)}{\text{norm}(Te - \text{Te})} \times 100
\] (7)

where \( Ts \) is simulated temperature, \( Te \) is measured temperature and \( \text{Te} \) is mean of temperature. The loss function \( V \) is follow in Eq. (8) where \( N \) is the number of estimation data and \( \theta_N \) is represents the estimated parameters:

\[
V = \det \left( \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t, \theta_N) (\varepsilon(t, \theta_N))^T \right)
\] (8)
Figure 3. T2, T3 and T4 temperature changes to given a step effect to the heater capacity between % 10-70 values [14]

1) Comparison Studies Between Experimental and NARX Models

Shown in Figure 4-6 were the comparison between the experimental and the simulated wavelet network NARX and sigmoid network NARX model outputs for T2-T3-T4 temperatures of the process simulator, respectively. As can be seen from Figure 4 there was a good agreement between the experimental and wavelet network NARX model simulated T2 temperature but there was a low agreement between the experimental and sigmoid network NARX model according to wavelet network NARX model. From the Figure 5, the wavelet network NARX and sigmoid network NARX model simulated profiles of T3 were similar to each other and these two NARX models had lower fit with experimental temperature profiles according to T2. From the Figure 6 there was a good agreement between the experimental and wavelet network NARX model simulated T4 temperature while there was a very low agreement between the experimental and sigmoid network NARX model.

In Figure 4-5 there were good agreement between experimental and simulated these two NARX models temperature profiles in first 300s and these two NARX models temperature profiles reaches steady state rapidly without any oscillations after 300s.

From the Table 1, it was observed that the fit values of the wavelet network NARX model which was calculated for T2, T3 and T4 were higher than that of the sigmoid network NARX model. It has been revealed from Table 1 that, even though the differences of the fit values were not too much between the T2, T3 and T4 temperatures for sigmoid network NARX model. According to the Table 1, fit values of the wavelet network NARX model decreased while fit values of the sigmoid network NARX model was not changed from the T2 to T4. The highest and the lowest fit values are calculated for T2 and T4 with NARX models, respectively.

From the Table 2, it was observed that the loss function values of the wavelet network NARX model which was calculated for T2, T3 and T4 were lower than that of the sigmoid network NARX model. It has been revealed from Table 2 that, even though the differences between the loss function values of two NARX models were found to be very small. According to the Table 2, loss function values of the both NARX models were decreased from the T2 to T4. The highest and the lowest loss function values are calculated for T2 and T4 with NARX models, respectively.
Table 1. Fit values (%) of the NARX models for T2, T3 and T4 temperatures

<table>
<thead>
<tr>
<th>Model</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelet network</td>
<td>87.66</td>
<td>82.27</td>
<td>77.49</td>
</tr>
<tr>
<td>sigmoid network</td>
<td>73.08</td>
<td>72.22</td>
<td>72.24</td>
</tr>
</tbody>
</table>

Table 2. Loss function values of the NARX models for T2, T3 and T4 temperatures

<table>
<thead>
<tr>
<th>Model</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelet network</td>
<td>0.018443</td>
<td>0.003114</td>
<td>0.002415</td>
</tr>
<tr>
<td>sigmoid network</td>
<td>0.019249</td>
<td>0.003673</td>
<td>0.002465</td>
</tr>
</tbody>
</table>

Figure 4. Comparison of experimental and two NARX model outputs for T2

Figure 5. Comparison of experimental and two NARX model outputs for T3
2) **NARX Model Simulation Studies**

In order to have ideas about the dynamics of the developed models, the NARX models were simulated with the aid of MATLAB by applying a step effect to each of them and their dynamic responses were recorded. The obtained dynamic responses for the wavelet network NARX and sigmoid network NARX models are as shown in Figures 7, 8 and 9 for temperatures T2, T3, and T4, respectively. From Figure 7, 8 and 9 for temperature T2, T3 and T4 these two NARX models step response results were stable because they were able to the steady state successfully. Wavelet network NARX model simulation results from Figure 7 and 8 for temperature T2 and T3 were stable because they were able to the steady state rapidly according to sigmoid network NARX results. In these three figures wavelet network NARX and sigmoid network NARX models results were similiar to each other without any oscillations before the steady state. In Figure 7 wavelet network NARX model getting steady state at 750s while sigmoid network NARX model getting steady state at 2200s. In Figure 8 wavelet network NARX model getting steady state at 3500s while sigmoid network NARX model getting steady state at 4500s. Based on the simulation results, wavelet network NARX models were discovered to be better than sigmoid network NARX models due to fastest rise time and fastest response time than sigmoid network NARX model results for this wireless control system.
IV. CONCLUSION

In this study the comparison of two types of nonlinear models (wavelet network NARX and sigmoid network NARX) for a process simulator has been carried out. Wireless temperature experiments were achieved by using MATLAB/Simulink program and wireless data transfer during the experiments were carried out using radio waves at a frequency of 2.4 GHz. The wireless data used for the two NARX models development were generated using a unit step change between % 10-70 values on the heater capacity of the process simulator. The comparisons between the wireless experimental and the simulated values of the temperatures (T2-T3-T4) of the wavelet network NARX and sigmoid network NARX models for the process simulator have shown that the wavelet network NARX model can be used to represent the behavior of the this wireless control system. According to the higher fit value and lower loss function observed in the case of the NARX model developed using wavelet network NARX model has been found to be better than sigmoid network NARX model. Based on the simulation results, wavelet network NARX models were discovered to be better than sigmoid network NARX models due to fastest rise time and fastest response time and both models in getting to the steady-state without any oscillations for this wireless control system.
V. ACKNOWLEDGEMENTS

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Nomenclature

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>MDL</td>
<td>Rissanen’s Minimum Description Length</td>
</tr>
<tr>
<td>ARX</td>
<td>AutoRegressive with eXogenous</td>
</tr>
<tr>
<td>NARX</td>
<td>Nonlinear AutoRegressive with eXogenous</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi Input Multi Output</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>R</td>
<td>Heater Capacity (%)</td>
</tr>
<tr>
<td>t</td>
<td>Time (s)</td>
</tr>
<tr>
<td>T₁</td>
<td>Storage tank temperature (°C)</td>
</tr>
<tr>
<td>T₂</td>
<td>Heater output temperature (°C)</td>
</tr>
<tr>
<td>T₃</td>
<td>Second tank input temperature (°C)</td>
</tr>
<tr>
<td>T₄</td>
<td>Second tank output temperature (°C)</td>
</tr>
</tbody>
</table>

REFERENCES
