Multi objective optimization of the power system by using 
ε-constraint method

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Abstract: This paper present enhance the power system operation by using ε-constraint method, for enhance the multi objectives are fuel cost, power loss, and loadability, for optimise the objective function use the PSO algorithm, the software for optimization is the MATLAB, Results for the IEEE-14 bus system and IEEE-30 bus system

Keywords: ε-constraint method, PSO algorithm, optimization

I. INTRODUCTION

In the olden days on words economic dispatch problem [1]-[4] as the main role in the power system optimal operation, it has some equality and inequality constraints for the optimal generation cost of the generating plants for the allocation, present public has more awareness on the pollution for the thermal power plant, by using economic dispatch not only return the income [5][6] it also reduce the pollution for the effective operation,

Different methods will be applied up to now optimize multi objective into single objective, here consider the direct conventional method for optimization emission as a constraint, but this method fail to optimise cost and emission at the same time, after a linear programming optimization method is used to solve EED problem to take one object at a time, but this method also failed to optimize cost and emission at the same time, in the other research work Zahani and Eisenberg convert multi objective into single objective by linear combination of different objective as a weighted sum[7],

It gets the inferior solution by the weighted sun method; now presently use the ε-constraint method for multi object optimization,

For solve the single objective straight forward method to get the direct solution, but the multi objective it get different solution it has some confusion to overcome that one we are using ε-constraint method, in that solve the one objective function consider reaming objective function as a constraint, by using pareto set we did not get the result without comparison, for that one compare the result with the other objective as constraint

For solve the multiobjective problem the ε-constraint method as more flexibility the weighting method [11], it control the density of pareto set for optimal solution and the most preferred solution is calculated by using fuzzy decision making tool, this method is applied to the IEEE30 bus system and IEEE-14 bus system for optimize the fuel cost, power loss, system loadability the future scope is the above objective function will be improved by placing the FACTS device

This paper is divided in the following section.

Section (2) problem formulation,
Section (3) constraint.
Section (4) PSO algorithm,
Section (5) problem statement
Section (6) MMP and algorithm
Section (7) result and conclusion
II. PROBLEM FORMULATION

Power system optimal operation can be optimize the power system steady-state performance of one or more objectives then satisfied some equality and inequality constraints, the objectives can be write in the following form

(I). Generation cost: the total generation cost is expressed as

\[
F_1(x) = \sum_{i=1}^{N_G} a_i P_{Gi}^2 + b_i P_{Gi} + c_i \quad \text{(}$$/\text{h})
\]

Where

\( P_{Gi} \) is the \( i^{th} \) generator active power

\( N_G \) is the total number of generators

\( a_i, b_i \) and \( c_i \) are generator constants

(ii). the total real power loss:

The total real power loss of the transmission lines is

\[
F_2(x) = \left\{ \sum_{k=1}^{nl} g_k \left[ V_i^2 + V_j^2 - 2V_iV_j \cos(\delta_i - \delta_j) \right] \right\}
\]

Where

\( nl \) is the total number of transmission lines;

\( g_k \) is the \( k^{th} \) line conductance;

\( V_i, V_j \) are the magnitude of the voltage of the buses \( i \) and \( j \)

\( \delta_i \) and \( \delta_j \) are the voltage angles of the buses \( i \) and \( j \)

(iii). Loadability: the Loadability of the system is

\[
F_3 = \rho(x,u)
\]

Where \( \rho \) is the loadability at each load in the real and reactive power balance equation (5) and (6)

III. CONSTRAINTS

For optimization of the objectives two sets of constraints that are equality and inequality constraints [10]

1. Equality constraints:

The equality constraints of the power system for the non-linear power flow equation for the active and reactive power is with respect to power balance equation
\[ P_G - \rho P_D = f_p(x,u) \]  \hspace{1cm} (5)  
\[ Q_G - \rho Q_D = f_q(x,u) \]  \hspace{1cm} (6)

Where the active and reactive powers are \( P_G \) and \( Q_G \); the load active and reactive powers are \( P_D \) and \( Q_D \); the real and reactive power flow equations are \( f_p \) and \( f_q \).

2. Inequality constraints:

The inequality constraints are

(a) Generator constraint:
\[ V_{Gi}^{\text{min}} \leq V_{Gi} \leq V_{Gi}^{\text{max}}, i = 1, \ldots, NG \]  \hspace{1cm} (7)
\[ P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}}, i = 1, \ldots, NG \]  \hspace{1cm} (8)
\[ Q_{Gi}^{\text{min}} \leq Q_{Gi} \leq Q_{Gi}^{\text{max}}, i = 1, \ldots, NG \]  \hspace{1cm} (9)

Where \( V_{Gi} \) is the voltage of the bus  
\( P_{Gi} \) is the real power of the \( i^{th} \) generator  
\( Q_{Gi} \) is the reactive power of the \( i^{th} \) generator  
\( NG \) is the total number of generators

(b) Transformer constraints:
\[ T_i^{\text{min}} \leq T_i \leq T_i^{\text{max}}, i = 1, \ldots, NT \]  \hspace{1cm} (10)

Where \( NT \) is the number of transformer

(c) Shunt VAR constraints:
\[ Q_{ci}^{\text{min}} \leq Q_{ci} \leq Q_{ci}^{\text{max}}, i = 1, \ldots, NC \]  \hspace{1cm} (11)

Where \( NC \) is the total number of shunt VAR compensation elements

(d) Security constraints:
\[ V_{Li}^{\text{min}} \leq V_{Li} \leq V_{Li}^{\text{max}}, i = 1, \ldots, NL \]  \hspace{1cm} (12)

Where \( NL \) is the total number of load buses
\[ S_i \leq S_i^{\text{max}}, i = 1, \ldots, nl \]  \hspace{1cm} (13)

Where \( nl \) is the total number of transmission lines

IV. PSO ALGORITHM [9]

1. Set the time counter \( t=0 \) and generate randomly \( n \) particles, \( \{Y_j(0), j = 1, \ldots, n\} \) where \( Y_j(0) = [Y_{j,1}(0), \ldots, Y_{j,n}(0)] \). \( Y_{j,k}(0) \) is generated by randomly selecting a value with uniform probability over the \( k^{th} \) optimized parameter search \([Y_{k,\text{min}}, Y_{k,\text{max}}]\). Similarly, generate randomly initial velocities of all particles \( \{U_j(0), j = 1, \ldots, n\} \) where \( U_j(0) = [U_{j,1}(0), \ldots, U_{j,n}(0)] \). \( U_{j,k}(0) \) is
generated by randomly selecting a value with uniform probability over the $k$th dimension $[-U_k^{\text{max}}, U_k^{\text{max}}]$. Each particle in the initial population is evaluated the objective function, $j$. For each particle, set $Y_j^*(0) = Y_j(0)$ and $J_j^* = J_j$, $j = 1, \ldots, n$. Search for the best value of the objective function $J_{\text{best}}$. Set the particle associated with $J_{\text{best}}$ as the global best. $Y^{**}(0)$, with an objective function $J^{**}$ as the initial value of the inertia weight $w(0)$.

2. (Time updating): update the time counter $t = t + 1$
3. (Weight updating): update the inertia weight $w(t) = \alpha w(t - 1)$
4. (Velocity updating): using the global best and individual best of each particle, the $j$th particle velocity in the $k$th dimension is updated according to the following evolution:

$$U_{j,k}(t) = w(t)U_{j,k}(t - 1) + c_1 r_1 (Y_{j,k}^*(t - 1) - Y_{j,k}(t - 1))$$
$$+ c_2 r_2 (Y_{j,k}^{**}(t - 1) - Y_{j,k}(t - 1))$$

where $c_1$ and $c_2$ are positive constants and $r_1$ and $r_2$ are uniformly distributed random members in $[0,1]$. It is worth mentioning that the second term represents the cognitive part of PSO where the particle changes its velocity based on its own thinking and memory, the third term represents the social part of PSO where the particle changes its velocity based on the social-psychological adaptation of the knowledge. If a particle violates the velocity limits, set its velocity equal to the limit.

5. (Position updating): based on the updated velocities, each particle changes its position according to the following equation:

$$Y_{j,k}(t) = U_{j,k}(t) + Y_{j,k}(t - 1)$$

If a particle violates the its position limits in any dimension, set its position at the proper limit.

6. (Individual best updating): each particle is evaluated according to its updated position, if $J_j < J_j^*$, $j = 1, \ldots, n$, then update individual best as $Y_j^*(t) = Y_j(t)$ and $J_j^* = J_j$, and go to step 7; else go to step 6.

7. (Global best updating): search for the minimum value $J_{\text{min}}$ and $J_{\text{min}}^*$, where min is the index of the particle with minimum objective function, i.e. $\text{min} \in \{j; j = 1, \ldots, n\}$ if $J_{\text{min}} < J^{**}$, then update global best as $Y^{**}(t) = Y_{\text{min}}(t)$ and $J^{**} = J_{\text{min}}$ and go to step 8; else go to step 6.

8. (Stopping criteria): if one of the stopping criteria is satisfied then stop; else go to step 2.

V. PROBLEM STATEMENT

The mathematically problem can be formulated as

Minimize $F_1$
Minimize $F_2$
Maximize $F_3$

Subject to some equality and inequality constraints

VI. MULTIOBJECTIVE MATHEMATICAL PROGRAMMING
There is no single solution for the optimization of the multi objective function at the same time, for that one we proposed one method that is $\varepsilon$-constraint method for multi objective mathematical programming and then generate pareto solutions, calculate the best pareto optimal solution by using fuzzy decision making tool, here discuss the both $\varepsilon$-constraint and fuzzy decision making tool.

A. $\varepsilon$-Constraint Method:[8]

In the $\varepsilon$-constraint method optimize one objective function consider other objective are the constraints

\[
\text{Min } F_1(x) \\
\text{Subject to } F_2(x) \leq e_2 F_1(x) \leq e_3 \ldots \ldots \ldots F_p(x) \leq e_p
\]

Where $p$ is the number of objective functions, to handle this method, $p-1$ objective functions will be used as constraint for the range of the objective function, by using payoff table to calculate the objective range (the table with the results from the individual optimization of the $p$ objective function), the individual optima of the objective functions are calculated to contract the payoff table (In the table $F_i^*$ indicate the optimal value of the $i^{th}$ objective function) where the value of other objective function is computed which are

\[
F_1^i, \ldots , F_{i-1}^i, F_{i+1}^i, \ldots , F_p^i, \text{the } i^{th} \text{ rows of the payoff table contains } F_1^i, \ldots , F_{i-1}^i, F_i^*, F_{i+1}^i, \ldots , F_p^i \text{ if calculated all the rows of the payoff table, from the payoff table to get the range of } k^{th} \text{ objective function from the } j^{th} \text{ column maximum and minimum values, that range divided into } (q_j-1) \text{ equidistant intermediate grid points, a total number } (q_i+1) \text{ grid points for the } j^{th} \text{ objective function, here total number sub problem for optimization is } (q_2+1) \times (q_3+1) \times \ldots \times (q_p+1).\text{the value assigned to the } q_i \text{ is represent the density of the optimal pareto set, the number interval for the objective function is selected to 4, in order deal the power system operation with the MMP problem, four objective functions } F_1, F_2, F_3 \text{ and } F_4 \text{ are considered, described in (1)-(4), the optimization sub problems are}
\]

\[
\text{min } F_1(x) \\
\text{subject to } F_2(x) \leq e_2, \quad (14) \\
F_3(x) \leq e_3, \\
e_{2,i} = \max(F_2) - \left( \frac{\max(F_2) - \min(F_2)}{q_2} \right) \times i, \quad (15) \\
i = 0, 1, \ldots , q_2 \\
e_{3,j} = \min(F_3) + \left( \frac{\max(F_3) - \min(F_3)}{q_3} \right) \times j, \quad (16) \\
j = 0, 1, \ldots , q_3 
\]

B. Fuzzy Decision-Making Tool
For optimize the multi objective to obtained the pareto optimal solution by solve the optimization subproblems, then decision maker needs a choice for getting best pareto solution for that we use the fuzzy decision making in that one calculate the membership function for the each objectives, for minimizing the objective the member ship function is

$$\mu_i^n = \begin{cases} 
1 & F_i^n \leq \min(F_i) \\
\max(F_i) - F_i^n & \min(F_i) \leq F_i^n \leq \max(F_i) \\
0 & F_i^n \geq \max(F_i) 
\end{cases} \quad (18)$$

$$i = 1, 2, 3$$

For maximize the objective the member ship function is

$$\mu_i^n = \begin{cases} 
0 & F_i^n \leq \min(F_i) \\
F_i^n - \min(F_i) & \min(F_i) \leq F_i^n \leq \max(F_i) \\
1 & F_i^n \geq \max(F_i) 
\end{cases} \quad (18)$$

$$i = 4$$

Where $F_i^n$ and $\mu_i^n$ are the $i$th objective function in the nth pareto optimal solution and membership function, the membership function for the best pareto optimal solution

$$\mu_{opt} = \max \left\{ \sum_{i=1}^{p} \left( \sum_{k=1}^{M} w_k \mu_{i,k} \right) \right\} \quad w_i \geq 0, \sum_{i=1}^{p} w_i = 1$$

$i^{th}$ Objective function weight value is $w_i$, $M$ is the number of pareto optimal solutions, the sum of weight is equal to one

**VII. CASE STUDIES [12]**

Results of Single-Objective and Multiobjective Optimizations for the IEEE 14-Bus System and IEEE-30 Bus System

<table>
<thead>
<tr>
<th>CASE</th>
<th>PARAMETERS</th>
<th>IEEE-14 BUS</th>
<th>IEEE-30 BUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Total fuel cost($/h)</td>
<td>19143.22</td>
<td>743.991</td>
</tr>
<tr>
<td></td>
<td>Ploss(MW)</td>
<td>6.042014</td>
<td>12.60201</td>
</tr>
<tr>
<td>F2</td>
<td>Ploss(MW)</td>
<td>1.238714</td>
<td>953.1983</td>
</tr>
<tr>
<td></td>
<td>Total fuel cost($/h)</td>
<td>20049.03</td>
<td>3.227887</td>
</tr>
<tr>
<td>F3</td>
<td>Loadability</td>
<td>1.40</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>Total fuel cost($/h)</td>
<td>34550.84</td>
<td>825.4946</td>
</tr>
<tr>
<td></td>
<td>Ploss(MW)</td>
<td>6.03947</td>
<td>29.96708</td>
</tr>
</tbody>
</table>
The results of single objective and multi objective optimizations are obtained and compared for the objective functions. The results of the single objective optimization are presented in Table 1. In order to study the conflict among the objective functions; the multi objective optimization is performed in four cases. In case 1, the total fuel cost and the power losses are minimized. The total fuel cost and the system loadability are optimized in Case 2. In case 3, the system loadability is studied to optimize the power losses and the system loadability. The total fuel cost, the power losses, the same weight Values are assigned to the objective functions. According to the adopted weighting factors, the most preferred compromise solution is selected among the Pareto optimal solutions using the fuzzy decision-making process in each case. The results of multi objective Optimizations are shown in Table. Some interesting observations which can be derived from this table are as follows: According to the single objective optimization, the results of mode 3 demonstrate a decrease in the total fuel cost and power losses as well as an increase in the system loadability compared to that of mode 1 while the results of mode 2 are in contrast point to those of mode 3. The

<table>
<thead>
<tr>
<th>Optimized Objectives</th>
<th>Two Objective functions</th>
<th>IEEE-14 BUS</th>
<th>IEEE-30 BUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1&amp;F2</td>
<td>Cost, Loss</td>
<td>17007.19</td>
<td>864.5511</td>
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<tr>
<td></td>
<td></td>
<td>1.288488</td>
<td>4.7784</td>
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<tr>
<td>F2&amp;F3</td>
<td>Loss, Loadability</td>
<td>10.43516</td>
<td>10.84085</td>
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<tr>
<td></td>
<td></td>
<td>1.39</td>
<td>0.28903</td>
</tr>
</tbody>
</table>

**REFERENCES**