USING HYBRID FUZZY ENTROPY FRAMEWORK FOR EVALUATION OF RELATIVE LEAN PERFORMANCE OF FIRMS
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Abstract-Evaluation of the lean status is very crucial for firms in resource constrained manufacturing environments that they operate in today. The information regarding the extent of lean adaptation achieved and relative lean status offers the producing units strong reasons to review the tactical and strategic aspects of their operations and reconfigure their systems in their drive to be/remain globally competitive. Several articles published hitherto have addressed the issue of leanness assessment but the search for even more objective and comprehensive evaluation methods is still on. In this paper, we propose Chang’s entropy based fuzzy AHP method for assessing leanness in MCDM environment. The method can deal in a natural manner with complexities inherent in the decision making environment due to imprecision or subjectivity of information. The method employs fuzzy set theory concepts for determination of criteria weights and information entropy approach for ranking the alternatives under considerations. A numerical example is provided to explain, illustrate and verify the practical application and usefulness of the approach. Sensitivity analysis is also carried out to study how certain subjective considerations influence the results. It can be concluded that the suggested method is quite versatile and efficient for solving wide array of MCDM applications.

Keywords: α-cut, Entropy, Lean thinking, MCDM, Sensitivity analysis

I. Introduction

Lean thinking aims at producing high quality products and services at the lowest cost with maximum customer responsiveness through systemic identification and elimination of waste. The term 'lean production' was first used by Krafcik (1988). It was first developed by Toyota, and adapted by others later. Lean manufacturing can be defined as a set of practices focused on reduction of wastes and non-value added activities from a firm’s manufacturing operations (Womack et al., 1990; McLachlan, 1997; Shah and Ward, 2003, 2007; Li et al., 2005; Browning and Heath, 2009). It focuses on the systematic elimination of wastes from an organization’s operations through a set of synergistic work practices to produce products and services at the rate of demand (Womack et al., 1990; Fullerton et al., 2003; Simpson and Power, 2005; Shah and Ward, 2007). Lean operations are characterized by the elimination of wastes occurring in the manufacturing process, thereby facilitating cost reduction (Serrano et al. 2008). Leanness should not be viewed merely as a set of tools, technique and practices (Papadopoulos and Zbayrak, 2005) but as a holistic approach that crosses the boundaries of shop floor to affect all the operational aspects of the organization.

The lean has a number of benefits to offer, including shorter cycle time, shorter lead times, lower WIP, faster response time, lower cost, greater production flexibility, higher quality, better customer service, higher revenue, higher throughput, and increased profit (Bhasin 2011). In their survey-based research, Wong et al. (2009) found that the benefits accruing to firms from lean implementation include improved flexibility (43.18%), improved response time (52.27%), improved quality (59.09%), increased profit (61.36%), decreased inventory (68.18%), reduced waste (81.82%), improved productivity (86.36%) and reduced cost (88.64%). The lean adaptation as a business improvement initiative has led
to a sharper focus on value adding processes that create customer value and close monitoring of critical processes (Ahmed, 2010).

A company wishing to go lean must have proper performance measurement system in place to quantify the efficiency and/or effectiveness of its actions and initiatives. Evaluation process outcomes provide the organizations with handy information that they can use to upgrade their systems and strengthen their competitive position significantly. All major businesses around the world have started trying to adopt lean in order to remain competitive in the increasingly global market (Pe´rez and Sa´nchez, 2000; Schonbergerm, 2007; Womack, et al., 1990).

Table 1

<table>
<thead>
<tr>
<th>Lean Issue</th>
<th>Sub-issues</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer issues (C2)</td>
<td>Frequent design and supply schedule changes, Reduced response time, Reduced rejection rate, delivery in small lots, implementation of self-certification, wide range of products</td>
<td>Rother and Shook (1999), Sim and Rogers (2009), McDonald et al. (2002), Singh and Sharma (2009),</td>
</tr>
</tbody>
</table>

Decision making in complex environments requires consideration and optimization of a number of factors at the same time and the methods employed to solve such MCDM problems can be broadly classified into two categories (Wang and Lee, 2007): Classical methods, where weights and ratings are given in crisp numbers and fuzzy methods, where linguistic terms are used to assign weights and rate performances. A cursory examination of literature shows that a good number of books and articles have been published on lean philosophy but academia and industry, the twain are still far from agreeing on or arriving at one best method of evaluating leanness (Shah and Ward 2007). In the present study, we
propose a hybrid framework for evaluating the relative leanness of firms of almost same size representing the same industrial segment. The framework utilizes fuzzy criteria weights and employs information entropy method for ranking the alternatives under considerations. The selection of attributes that serve as criteria for any evaluation problem is very important and the attributes chosen must represent the overall measure of the performance. According to Neely et al. (1997) ill-designed performance indicators can result in dysfunctional behaviors, encouraging individuals to make the wrong decisions. In this study, the lean evaluation criteria selection is made by identifying various lean sub-issues through literature review and then categorizing them into five main lean issues (Table 1) that serve as criteria. Sensitivity analysis is carried out to know the effect of variation in degree of optimism on the results obtained from the proposed framework. The findings and states some of the limitations of the study. This section also suggests new directions for future research in the area.

II. Literature Review

The review of the extant literature on lean performance measurement efforts shows that several approaches have been adopted by researchers for assessing lean performance of firms. A brief account of the previous work in this area is presented here.

Vinodh and Kumar (2012) developed a decision support system for multi grade fuzzy leanness assessment (DSS-MGFLA). Vinodh and Vimal (2011) designed a conceptual model for leanness assessment and developed a fuzzy leanness index which indicates the leanness level of the organization. Vinodh and Balaji (2011) developed a computerized fuzzy logic based leanness assessment decision support system (FLBLA-DSS) which computes the fuzzy leanness index and identifies the weaker areas which need improvement. Zarei et al. (2011) presented a framework based upon expert system to evaluate the Leanness Achievement Degree (LAD) of organizations. Behrouzi and Wong (2011) presented an innovative approach to measure the lean performance of manufacturing systems with the help of fuzzy membership functions. Saurin et al. (2011) drew up a framework for assessing the use of lean production practices in manufacturing cells based on inter-relationships among those practices. Seyedhosseini et al. (2011) developed the concept of Balanced scorecard (BSC) approach for choosing the leanness criteria. Singh et al. (2010) developed a leanness index for measuring leanness of an auto component industry using fuzzy theory concepts. Gurumurthy and Kodali (2009) utilized structured benchmarking to help an organization know where it stands vis-à-vis other lean manufacturing (LM) organizations. Bhasin (2008) adapted a Dynamic Multi-dimensional Performance (DMP) model which enables organizations to gauge, in a holistic manner, whether lean has in fact proven successful in their respective organizations. Bayou and de Korvin (2008) used the fuzzy-based methodology to compare leanness of production systems of General Motors and Ford Corporation. Wan and Frank (2008) proposed a unit-invariant leanness measure with a self-contained benchmark to quantify the leanness level of manufacturing systems. Shah and Ward (2007) developed an operational measure of lean production in the form of a framework that identifies salient dimensions of lean production. Srinivasaraghavan and Allada (2006) used Mahalanobis distance, calculated by the Gram-Schmidt Orthogonalization process to objectively measure the lean efforts of an organization. Kojima and Kaplinsky (2004) carried out a study in South African auto components sector regarding various issues of lean production and proposed a lean production index (LPI) based on flexibility, continuous improvement, and quality dimensions to measure degree of progress in adaptation of lean production. Multi-grade fuzzy approach has been used for computation of leanness (Yang and Li, 2002).

Meier and Forrester (2002) developed and tested a model that could evaluate the degree of leanness possessed by manufacturing firms using a number of lean issues such as elimination of waste, continuous improvement, zero defects, just in time deliveries, pull of materials, multifunctional team, decentralization, integration function and vertical information.
\section*{III. Fuzzy set theory preliminaries}

Many real world decision making problems cannot be described and handled by classical set theory as the level of preference cannot be precisely and adequately defined using crisp numerical values. Such imprecision may arise from a variety of reasons like unquantifiable, incomplete or unobtainable information and partial ignorance. The conventional MCDM methods cannot effectively deal with problems encumbered with such imprecise information. The results of real-world decision making problems can be misleading if the fuzziness of human decision making is not taken into account (Tsaur et al., 2002). Zadeh (1965) introduced the fuzzy set theory in order to deal with the instances of subjective/qualitative information.

The fuzzy set theory not only allows for more realistic representation of the real-world, but it also does so with simplicity. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill defined to be described in conventional quantitative terms (Zimmermann, 1991). A fuzzy set is an extension of a crisp set that allows for partial membership (Chen and Pham, 2001) and fuzzy linguistic variables are the variables whose values are not numbers, but lingual expression or terms (Wang, et al., 2009). The linguistic terms are intuitively easier to use when decision makers express the subjectivity and imprecision of their assessments (Herrera and Herrera-Viedma, 2000). A fuzzy set is characterized by a membership function which assigns a grade of membership to each element within the interval [0,1] that indicates the degree to which an element is a member of the set (Bevilacqua, Ciarpica, & Giachetta, 2006). A fuzzy number can be thought of as a function whose domain is a specified set, usually of real numbers, and whose range is the span of non-negative real numbers between, and including, 0 and 1 (Zadeh, 1965). The triangular and trapezoidal fuzzy numbers are used in general ((Baykal and Beyan, 2004) but Triangular fuzzy numbers (TFNs) are preferred because of their computationally efficient information processing and representation in a fuzzy environment ((Karsak, 2002). Moreover, modeling using TFN has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise (Mergias et al., 2007). A triangular fuzzy number (Fig. 1) can be defined by a triplet \((l, m, u)\) to describe a fuzzy event (Deng, 1999; Mahdavi et al., 2008).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Graphic representation of a TFN}
\end{figure}

The parameters \(l, m\) and \(u\) indicate the smallest possible value, the most promising value, and the largest possible value respectively. The membership function \(\mu_A(x)\) is defined in the universe of discourse \([0, 1]\) (Zadeh, 1965) as,

\[
\mu_A(x) = \begin{cases} 
\frac{(x-l)}{(m-l)} & x \in [l, m] \\
\frac{(u-x)}{(u-m)} & x \in [m, u] \\
0 & Otherwise 
\end{cases}
\]

\subsection*{3.1. Operations on TFNs}
There are various operations on TFNs (Kaufmann and Gupta, 1988) but, only the ones used in this paper are illustrated here. Let two TFNs \( A \) and \( B \), be defined by the triplets \( A = (l_1, m_1, u_1) \) and \( B = (l_2, m_2, u_2) \) as shown in Fig. 2, then (Chien and Tsai, 2000):

(i) Addition: \( A + B = (l_1, m_1, u_1) + (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \),
(ii) Multiplication: \( A \times B = (l_1, m_1, u_1) \times (l_2, m_2, u_2) = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2) \)

3.2. Aggregation of Fuzzy Numbers

If the fuzzy ratings of all experts is described as a fuzzy number \( \tilde{R}_k = (l_k, m_k, u_k) \) \( k = 1,2,\ldots,K \), then the aggregate fuzzy rating is given by, \( \tilde{R} = (l, m, u) \), \( k = 1,2,\ldots,K \), where,

\[
\begin{align*}
    l &= \min \left\{ l_k \right\}, \\
    m &= \frac{1}{K} \sum_{k=1}^{K} m_k, \\
    u &= \max \left\{ u_k \right\}.
\end{align*}
\]

3.3. Determination of \( \alpha \) cut interval

An \( \alpha \)-cut of a fuzzy set \( A \) is a crisp set \( \bar{A}_\alpha \) that contains all the elements in \( X \) that have membership value in \( A \) greater than or equal to \( \alpha \) and is defined as (Zimmerman, 1991):

\[
\bar{A}_\alpha = \left\{ x | \mu_A(x) \geq \alpha, x \in X \right\}, 0 \leq \alpha \leq 1.
\]

For a TFN \( \tilde{A} = (l, m, u) \) shown in Fig. 3, the \( \alpha \) cut lower and upper bounds of the closed interval are found using interval arithmetic as:

\[
\bar{A}_\alpha = [l_\alpha, u_\alpha] = [l + (m - l)\alpha, u - (u - m)\alpha]
\]

IV. Entropy-based fuzzy AHP method

Chang’s (1996) developed this multi-criteria evaluation method by integrating the principle of maximum entropy (Shannon, 1948) with fuzzy analytical hierarchy process. Shannon’s entropy function expressed as,

\[
H(m) = -\sum_{j=1}^{n} m(\{x\}) \log_2 m(\{x\})
\]

Measures the average uncertainty associated with the prediction of outcomes in a random experiment (Klir and Yan, 1995). Its range is \([0, \log_2 |X|]\).
Clearly, \( H(m) = 0 \) when \( m(x_i) = 1 \) for some \( x_i \in X \); \( H(m) = \log_2 |X| \) where \( m \) defines the uniform probabilities distribution on \( X \) (i.e., \( (m(x_i)) = 1/|X|, \forall x_i \in X \)).

The computational procedure of this decision-making methodology is summarized as follows:

**Step 1:** Obtain the aggregate fuzzy subjective criteria weight vector. The fuzzy weights assigned to various criteria by the nominated experts are integrated using Eq. (2).

**Step 2:** Develop fuzzy evaluation matrix \( \tilde{x} \) based on experts’ linguistic assessment of performance of selected firms. The linguistic variables are parameterized by assigning them triplet.

**Step 3:** Assemble the total fuzzy judgment matrix \( \tilde{A} \) by multiplying the fuzzy subjective weight Vector \( \tilde{W} \) with the corresponding column of fuzzy judgment matrix \( \tilde{x} \). Thus, we get,

\[
\tilde{A} = \begin{bmatrix}
\tilde{w}_1 \otimes \tilde{x}_{11} & \tilde{w}_2 \otimes \tilde{x}_{12} & \cdots & \tilde{w}_m \otimes \tilde{x}_{1m} \\
\tilde{w}_1 \otimes \tilde{x}_{21} & \tilde{w}_2 \otimes \tilde{x}_{22} & \cdots & \tilde{w}_m \otimes \tilde{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{w}_1 \otimes \tilde{x}_{n1} & \tilde{w}_2 \otimes \tilde{x}_{n2} & \cdots & \tilde{w}_m \otimes \tilde{x}_{nm}
\end{bmatrix}
\]

**Step 4:** Set the value of \( \alpha \) and make \( \alpha \)-cuts using Eq. (3). The \( \alpha \)-cut matrix, \( \tilde{A}_\alpha \) for various Alternative - criteria combinations is thus obtained as:

\[
\tilde{A}_\alpha = \begin{bmatrix}
[a_{11}^\alpha, a_{11}^\alpha] & \cdots & [a_{1m}^\alpha, a_{1m}^\alpha] \\
\vdots & \ddots & \vdots \\
[a_{n1}^\alpha, a_{n1}^\alpha] & \cdots & [a_{nm}^\alpha, a_{nm}^\alpha]
\end{bmatrix}
\]

Where, \( a_{ij}^\alpha = w_{ij}^\alpha x_{ij}^\alpha, a_{mn}^\alpha = w_{mn}^\alpha x_{mn}^\alpha \) for \( 0 < \alpha \leq 1 \) and all \( i,j \).

**Step 5:** Construct the precision judgment matrix and estimate the degree of satisfaction of the Judgment \( \tilde{A} \). For a fixed value of \( \alpha \), the index of optimism \( \lambda \) is set by the degree of the optimism of a decision maker. A larger \( \lambda \) indicates a higher degree of optimism. The index of optimism is a linear convex combination and is explained by

\[
\hat{a}_{ij} = (1 - \lambda)a_{ij}^\alpha + \lambda a_{mn}^\alpha, \forall \lambda \in [0, 1]
\]

The precise judgment matrix \( \tilde{A} \) is thus obtained as:

\[
\tilde{A} = \begin{bmatrix}
\hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\
\hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{a}_{n1} & \hat{a}_{n2} & \cdots & \hat{a}_{nn}
\end{bmatrix}
\]

**Step 6:** Calculate the relative frequency using the relation,

\[
s_k = \sum_{j=1}^{n} a_{kj}
\]
Step 7: Compute the entropy values of all the alternatives as per the equation given below. The Entropy value for the \( i^{th} \) alternative, \( H_i \), is computed as:

\[
H_i = -\sum_{j=1}^{n} \left( f_{ij} \right) \log_2 \left( f_{ij} \right)
\]

Step 8: The entropy weights of are determined by normalizing the \( H_i \) values as follows:

\[
W_i = \frac{H_i}{\sum_{i=1}^{n} H_i}
\]

Illustrative example

This section first presents an overview of the decision making problem and then describes its step by step solution procedure. Three experts are nominated and their linguistic assessment of the predefined evaluation criteria and firms’ lean performance on a five point scale is solicited. The linguistic scale (Fig. 4) chosen for assessing criteria importance and firms’ performance on various lean criteria has five levels; Very Low / Very Poor \((1,1,3)\); Low / Poor \((1,3,5)\); Medium / Fair \((3,5,7)\), High / Good \((5,7,9)\) and Very High / Very Good \((7,9,9)\).

![Fig.4 Membership function of five levels of linguistic variables](image)

Table 2 gives the linguistic ratings and parameterized values of criteria weights.

Fuzzy criteria weight

The aggregate fuzzy subjective criteria weight vector, \( W_{C1-C5} = [(3,7,67,9), (3,7,00,9), (3,5,67,9), (1,3,67,9), (1,5,67,9)] \) is obtained by integrating the fuzzy weights assigned to the five criteria by all the experts. Table 3 shows the linguistic fuzzy ratings of the firms assigned by the three experts to all firm-criteria combinations. It also provides the parameterized value of the linguistic variables and the aggregate fuzzy ratings for different firm-criteria combinations.
Table. 2
Fuzzy aggregate decision

<table>
<thead>
<tr>
<th>firm</th>
<th>Expert</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E₁</td>
<td>VG</td>
<td>G</td>
<td>(1,3,5)</td>
<td>VG</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>E₂</td>
<td>G</td>
<td>VG</td>
<td>(3,5,7)</td>
<td>G</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>E₃</td>
<td>F</td>
<td>P</td>
<td>(1,3,5)</td>
<td>G</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>Aggregated Value</td>
<td>(3,7,9)</td>
<td>(1,6.3 3,9)</td>
<td>1,5,9</td>
<td>(5,7, 67,9)</td>
<td>(1,4, 33,7)</td>
</tr>
<tr>
<td></td>
<td>E₁</td>
<td>P</td>
<td>F</td>
<td>(3,5,7)</td>
<td>G</td>
<td>VP</td>
</tr>
<tr>
<td></td>
<td>E₂</td>
<td>F</td>
<td>VP</td>
<td>(1,1,3)</td>
<td>VG</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>E₃</td>
<td>G</td>
<td>G</td>
<td>(1,3,5)</td>
<td>F</td>
<td>VG</td>
</tr>
<tr>
<td></td>
<td>Aggregated Value</td>
<td>(1,5,00 9)</td>
<td>(1,433,9)</td>
<td>1,5,67,9</td>
<td>(3,7, 9)</td>
<td>(1,4, 33,9)</td>
</tr>
<tr>
<td></td>
<td>E₁</td>
<td>G</td>
<td>G</td>
<td>(1,3,5)</td>
<td>F</td>
<td>VG</td>
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<tr>
<td></td>
<td>E₃</td>
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<td>VP</td>
<td>(1,1,3)</td>
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<td>F</td>
</tr>
<tr>
<td></td>
<td>Aggregated Value</td>
<td>(5,7,67 9)</td>
<td>(1,3,6 7,9)</td>
<td>1,5,9</td>
<td>(3,7, 9)</td>
<td>(1,5, 00,9)</td>
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<td></td>
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<td>F</td>
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<td>(1,3,5)</td>
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<td>F</td>
</tr>
<tr>
<td></td>
<td>Aggregated Value</td>
<td>(3,7,9)</td>
<td>(3,7,9 )</td>
<td>1,433,9</td>
<td>(1,4, 33,9)</td>
<td>(1,3, 67,7)</td>
</tr>
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</table>

The total fuzzy judgment matrix (Table 4) is obtained using Eq. (4).
Table 3

<table>
<thead>
<tr>
<th>Firm</th>
<th>Fα-cut</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>C2</td>
<td>C3</td>
</tr>
<tr>
<td>E1</td>
<td>V</td>
<td>V</td>
</tr>
<tr>
<td>H (7,9,9)</td>
<td>V</td>
<td>H (7,9,9)</td>
</tr>
<tr>
<td>C2</td>
<td>H (7,9,9)</td>
<td>H (5,7,9)</td>
</tr>
<tr>
<td>C3</td>
<td>M (3,5,7)</td>
<td>M (3,5,7)</td>
</tr>
<tr>
<td>C4</td>
<td>H (5,7,9)</td>
<td>L (1,3,5)</td>
</tr>
<tr>
<td>C5</td>
<td>H (5,7,9)</td>
<td>H (5,7,9)</td>
</tr>
</tbody>
</table>

Table 4

Lean Issue | E1 | E2 | E3
---|---|---|---
C1 | V | V | M
H (7,9,9) | V | H (7,9,9) | (3,5,7)
C2 | H (7,9,9) | H (5,7,9) | M
C3 | M (3,5,7) | M (3,5,7) | H
C4 | H (5,7,9) | L (1,3,5) | L
C5 | H (5,7,9) | H (5,7,9) | L

Table 5

Total fuzzy judgment matrix

<table>
<thead>
<tr>
<th>Firm</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>C2</td>
</tr>
<tr>
<td>E1</td>
<td>E2</td>
</tr>
<tr>
<td>(3,7,67,9)</td>
<td>(3,7,00,9)</td>
</tr>
<tr>
<td>(3,7,00,9)</td>
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<td>(3,7,00,9)</td>
<td>(1,5,00,9)</td>
</tr>
</tbody>
</table>
The sample calculations for computing $\alpha$-cut and precise judgment values for Firm 1 - Criterion $C_1$ combination are shown below. For, $\alpha = 0.8$, the $\alpha$-cut value, is determined using Eq. (3) as

$$[\alpha^1_{\alpha}, \alpha^1_{\alpha}] = [(7.67-3) \times 0.8 + 3, -(9.767) \times 0.8 + 9] \otimes [(7.67-3) \times 0.8 + 3, -(9.767) \times 0.8 + 9]$$

$$= [41.76, 58.73]$$

The precise judgment value for index of optimism $\lambda = 0.5$ is computed using Eq. (5) as:

$$\tilde{\alpha}^1_1 = [(1-0.5) \times 41.76 + 0.5 \times 58.73] = 50.24.$$
So, the ranking order obtained using the proposed framework is: Firm 2 > Firm 4 > Firm 1 > Firm 3 > Firm 5.

V. Sensitivity analysis

Sensitivity analysis lets us know the impact of change in some input variables on the overall results yielded by methodology adopted for solution. It gives a fairly good idea about the stability and consistency of the solution methodologies. In this study, sensitivity analysis is carried out for observing the effect of change in index of optimism and criteria weights on the ranking order generated by the proposed entropy based fuzzy AHP method.

5.1. Effect of variation in \( \lambda \) on ranking order

The degree of optimism adopted by the decision makers in introduces an additional element of subjectivity in evaluations. The change in the value of \( \lambda \) only tends to push the precise judgment value towards upper or lower bound of \( \alpha \)-cut interval. The precise judgment values for all the alternatives change in same proportion effecting no change in their relative position. Here, sensitivity analysis is carried out by conducting ten experiments wherein the value of \( \lambda \) is varied from 0 to 1 in steps of 0.1, skipping \( \lambda = 0.5 \) which has already been considered in the current study. Fuzzy entropy steps are then performed with these different scenarios and the resulting changes in the final ranking of the alternatives are observed.

Table 8
Effect of varying index of optimism on ranking order

<table>
<thead>
<tr>
<th>Expt. no.</th>
<th>Index of optimism</th>
<th>Ranking order (Entropy weights)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current study</td>
<td>0.5</td>
<td>F2 (0.2037) &gt; F4 (0.2032) &gt; F1 (0.2002) &gt; F3 (0.1987) &gt; F5 (0.1942)</td>
</tr>
<tr>
<td>Expt. 1</td>
<td>0.0</td>
<td>F2 (0.2056) &gt; F4 (0.2052) &gt; F1 (0.2008) &gt; F3 (0.1970) &gt; F5 (0.1914)</td>
</tr>
<tr>
<td>Expt. 2</td>
<td>0.1</td>
<td>F2 (0.2051) &gt; F4 (0.2047) &gt; F1 (0.2006) &gt; F3 (0.1974) &gt; F5 (0.1922)</td>
</tr>
<tr>
<td>Expt. 3</td>
<td>0.2</td>
<td>F2 (0.2047) &gt; F4 (0.2042) &gt; F1 (0.2005) &gt; F3 (0.1979) &gt; F5 (0.1928)</td>
</tr>
<tr>
<td>Expt. 4</td>
<td>0.3</td>
<td>F2 (0.2043) &gt; F4 (0.2038) &gt; F1 (0.2004) &gt; F3 (0.1982) &gt; F5 (0.1933)</td>
</tr>
<tr>
<td>Expt. 5</td>
<td>0.4</td>
<td>F2 (0.2040) &gt; F4 (0.2035) &gt; F1 (0.2003) &gt; F3 (0.1985) &gt; F5 (0.1938)</td>
</tr>
<tr>
<td>Expt. 6</td>
<td>0.6</td>
<td>F2 (0.2034) &gt; F4 (0.2029) &gt; F1 (0.2001) &gt; F3 (0.1990) &gt; F5 (0.1946)</td>
</tr>
<tr>
<td>Expt. 7</td>
<td>0.7</td>
<td>F2 (0.2032) &gt; F4 (0.2027) &gt; F1 (0.2001) &gt; F3 (0.1991) &gt; F5 (0.1949)</td>
</tr>
<tr>
<td>Expt. 8</td>
<td>0.8</td>
<td>F2 (0.2030) &gt; F4 (0.2025) &gt; F1 (0.2000) &gt; F3 (0.1993) &gt; F5 (0.1952)</td>
</tr>
<tr>
<td>Expt. 9</td>
<td>0.9</td>
<td>F2 (0.2028) &gt; F4 (0.2023) &gt; F1 (0.2000) &gt; F3 (0.1994) &gt; F5 (0.1954)</td>
</tr>
<tr>
<td>Expt. 10</td>
<td>1.0</td>
<td>F2 (0.2027) &gt; F4 (0.2021) &gt; F1 (0.2000) &gt; F3 (0.1996) &gt; F5 (0.1957)</td>
</tr>
</tbody>
</table>

The results of the analysis (Table 8) show that the ranking order for all the experiments is same irrespective of the index of optimism (\( \lambda \)) chosen.
<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Weight assignment</th>
<th>Ranking order (Entropy weights)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current Study</strong></td>
<td></td>
<td><strong>F2 (0.2037) &gt; F4 (0.2032) &gt; F1 (0.2002) &gt; F3 (0.1987) &gt; F5 (0.1942)</strong></td>
</tr>
<tr>
<td>t. 1</td>
<td>( W_{c1-c5} = (1, 1, 3) )</td>
<td><strong>F4 (0.2018) &gt; F2 (0.2007) &gt; F1 (0.1998) &gt; F3 (0.1991) &gt; F5 (0.1987)</strong></td>
</tr>
<tr>
<td>t. 2</td>
<td>( W_{c1-c5} = (1, 3, 5) )</td>
<td><strong>F4 (0.2018) &gt; F2 (0.2007) &gt; F1 (0.1998) &gt; F3 (0.1990) &gt; F5 (0.1986)</strong></td>
</tr>
<tr>
<td>t. 3</td>
<td>( W_{c1-c5} = (3, 5, 7) )</td>
<td><strong>F4 (0.2019) &gt; F2 (0.2007) &gt; F1 (0.1998) &gt; F3 (0.1990) &gt; F5 (0.1986)</strong></td>
</tr>
<tr>
<td>t. 4</td>
<td>( W_{c1-c5} = (5, 7, 9) )</td>
<td><strong>F4 (0.2019) &gt; F2 (0.2007) &gt; F1 (0.1998) &gt; F3 (0.1990) &gt; F5 (0.1986)</strong></td>
</tr>
<tr>
<td>t. 5</td>
<td>( W_{c1-c5} = (7, 9, 9) )</td>
<td><strong>F4 (0.2034) &gt; F3 (0.2029) &gt; F5 (0.1991) &gt; F1 (0.1977) &gt; F2 (0.1969)</strong></td>
</tr>
<tr>
<td>t. 6</td>
<td>( W_{c1} = (1, 1, 3) ), ( W_{c2-c5} = (7, 9, 9) )</td>
<td><strong>F3 (0.2018) &gt; F5 (0.2005) &gt; F4 (0.1998) &gt; F1 (0.1995) &gt; F2 (0.1984)</strong></td>
</tr>
<tr>
<td>t. 7</td>
<td>( W_{c2} = (1, 1, 3) ), ( W_{c1-c3} = (7, 9, 9) )</td>
<td><strong>F5 (0.2023) &gt; F4 (0.2015) &gt; F1 (0.1996) &gt; F2 (0.1995) &gt; F3 (0.1971)</strong></td>
</tr>
<tr>
<td>t. 8</td>
<td>( W_{c3} = (1, 1, 3) ), ( W_{c1-c2} = (7, 9, 9) )</td>
<td><strong>F4 (0.2025) &gt; F2 (0.2021) &gt; F1 (0.1992) &gt; F3 (0.1984) &gt; F5 (0.1978)</strong></td>
</tr>
<tr>
<td>t. 9</td>
<td>( W_{c4} = (1, 1, 3) ), ( W_{c1-c3} = (7, 9, 9) )</td>
<td><strong>F2 (0.2033) &gt; F1 (0.2015) &gt; F4 (0.2010) &gt; F3 (0.1992) &gt; F5 (0.1950)</strong></td>
</tr>
<tr>
<td>t. 10</td>
<td>( W_{c5} = (1, 1, 3) ), ( W_{c1-c4} = (7, 9, 9) )</td>
<td><strong>F4 (0.2048) &gt; F2 (0.2004) &gt; F1 (0.1990) &gt; F3 (0.1983) &gt; F5 (0.1975)</strong></td>
</tr>
</tbody>
</table>
5.2. Effect of variation in criteria weights on ranking order

A total of 10 experiments, the details of which are presented in Table 9, were conducted. The first case in the table represents set of weights computed in the current study \( W_1 = (3.00, 7.67, 9.00), \ W_2 = (3.00, 7.00, 9.00), \ W_3 = (3.00, 5.67, 9.00), \ W_4 = (1.00, 3.67, 9.00), \) and \( W_5 = (1.00, 5.67, 9.00) \). In the first five experiments equal weights are assigned to all criteria in the first five experiments, that is, the weights of all criteria are set equal to \((1,1,3)\) in the first experiment, then \( (1,3,5) \) in the second experiment and \( (3,5,7), (5,7,9) \) and \( (7,9,9) \) for the third, fourth and fifth experiment respectively. In the remaining four experiments, the weight combinations are chosen by setting, one by one, the weight of one criterion at the lowest level \((1,1,3)\) and that of the remaining at the highest value \((7,9,9)\) as shown in Table 9.

**Table 10**
Effect of variation in criteria weights on ranking order

The results of the analysis show that Firm 4 is on the top of the ranking order in seven out of 10 experiments. Table 10 shows the frequency of ranks of firms in the ten experiments.

**Table 10**
Frequency of ranks of firms

<table>
<thead>
<tr>
<th>Firm</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Firm</td>
</tr>
<tr>
<td>2</td>
<td>Firm</td>
</tr>
<tr>
<td>3</td>
<td>Firm</td>
</tr>
<tr>
<td>4</td>
<td>Firm</td>
</tr>
<tr>
<td>5</td>
<td>Firm</td>
</tr>
</tbody>
</table>

Six experiments, namely 1, 2, 3, 4, 8 and 10 yield the same ranking order \( F4 > F2 > F1 > F3 > F5 \)

**Conclusions**

This particular study proposes a scientific framework which uses a hybrid method that combines fuzzy set theory concepts with Shannon’s entropy theory for relative evaluation of lean performance of firms. The proposed approach is successfully applied and validated with the help of an illustrative example. The research provides the firms with handy information that they could use for benchmarking purpose. The analysis of comparative performance could suggest the firms the need for incorporating some changes in certain specific areas for ensuring improved utilization of and returns from available organizational resources.

The research findings can be summarized as follows: The firm with the highest entropy weight is the best lean performer. The order of ranking of the firms in our illustration is \( F2 > F4 > F1 > F3 > F5 \). The sensitivity analysis shows that the degree of optimism, \( \lambda \) adopted for arriving at precise judgment values of firms’ performance on various criteria does not have significant effect on the results yielded by the proposed methodology. The assignment of weights, however, has a telling effect on the
results. The frequency of ranks obtained by the firms (Table 10) suggests that the order of ranking should be modified slightly to: F4 > F2 > F1 > F3 > F5.

The perceptual errors can creep in due to subjectivity involved in expression of preferences and opinions by the experts and affect the quality of decisions. So, utmost care should be taken in selection of experts for assessments. As a future direction of research, the proposed approach with slight modifications can be applied to other MCDM situations in a similar environmental setting. The number of criteria and alternatives can also be varied. Moreover, similar studies can be conducted using other MCDM techniques such as; fuzzy VIKOR, fuzzy PROMETHEE, or fuzzy ELECTRE for comparative performance evaluation.

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