STUDY OF ADAPTIVE NEURAL CONTROLLER FOR NON-LINEAR SYSTEM USING RBFNN APPROXIMATION

Sahil I. Attar$^1$ and R.P. Hasabe$^2$

$^{1,2}$Department of Electrical Engineering, Walchand College of Engineering, Sangli-416415, India

Abstract— In this paper design method for adaptive neural controller is proposed and it is applied to the non-linear system CSTR. Controller is designed such way that it can adapt the process. To understand the effect of footprint of uncertainty on the controllers’ performance two types of techniques namely state feedback control and observer based control are designed. Radial basis function NN (RBFNN) is used for approximation of the nonlinear function.

Keywords— Adaptive neural controller, CSTR, radial basis function.

I. INTRODUCTION

It is a well-known that conventional PID controllers are the most common controllers used in industry because of their simple structure and low price [1]. The application of PID controllers in controlling linear system might be an effective way to achieve desired performance, but when the process model is uncertain or the process is non-linear, PID controllers might not achieve better result.

In the past several decades, development of new control strategies has been largely based on modern and classical control theories. Modern control theories such as adaptive and optimal control. Modern control theories such as adaptive and optimal control and classical control theories are mainly based on the linearization of system. In application point of view mathematical modeling is prior necessary. Robust control of nonlinear system is important topic in control system in both of the manners i.e. theoretically and practically, having attracted a lot of devotion in last few decades. It has been specified in [3]. There are two main limitations related to uncertainties in the nonlinear system. The first is the matching condition [4] and to control such type of the system some methods are proposed [5]. Second is triangularity assumption. Different methods have been used to control uncertain system with mismatched uncertainties [6]. Considering mismatched uncertainty is challenging problem since it can affect the system performance adversely. Some of the real applications can be seen in [7]. In recent years, considerable works have been made to design mismatched control systems with no assumptions on the bound of uncertainties [8]. Furthermore radial basis function neural network (RBFNN) is used many times for practical applications due to nice approximation properties and simple structure. RBF neural network is addressed few decades early but it is now taking much attention due to its good generalization ability and simple network structure that avoids unnecessary and lengthy calculations as compared to the multilayer feed forward network. Past research on universal approximation theorems on RBF have shown that any nonlinear function over compact set with arbitrary accuracy can be approximated by RBF neural network. The field of mathematical modeling, a radial basis function network is an artificial neural network that uses radial basis functions as activation functions. The output of the network is linear combination of radial basis function of the input and neuron parameters. Radial basis function network has many uses, including function approximation, time series prediction, classification and system control. In real world application there are many nonlinearities, unmodeled dynamics, and immeasurable noise which propose problems for engineers to apply control strategies.

Even though different methods are available for controlling the nonlinear system, most of the previous results are taken with the assumption that state variables are available. IF these states are not available, these results are not applicable in practice In this paper two adaptive neural
stabilization controllers for nonlinear system with uncertainties are proposed. The first controller is based on the state feedback and second is based on the estimated states.

1) This paper successfully extends adaptive neural controller for nonlinear system with uncertainties.
2) An observer is designed to estimate un-measurable states for control purpose.
3) By using robustifying term in control signal, the effects of approximation error in neural network is compensated.

II. PROBLEM FORMULATION

Consider the following nonlinear system with mismatched uncertainties.

\[ \dot{x} = Ax + f(x) + Bu \quad (1) \]

where \( x = [x_1 \ldots x_n]^T \in \mathbb{R}^n \) is the vector of system states, \( u = [u_1 \ldots u_m]^T \in \mathbb{R}^m \) is the vector of system inputs, \( f(x) = [f_1(x) \ldots f_n(x)]^T \in \mathbb{R}^m \) is the vector of smooth nonlinear mismatched uncertain functions.

In this paper, the Gaussian RBF will be employed to approximate a nonlinear function \( h(.) \) as follows:

\[ \hat{h}(z) = \Theta \xi(z) \quad (2) \]

where \( z \) is the input vector, \( \Theta = [\theta_1 \ldots \theta_n] \) is the weight vector, \( l \) is the number of nodes, \( \xi = [\xi_1 \ldots \xi_n]^T \in \mathbb{R}^m \) is the basis function determined priory, and commonly chosen as Gaussian functions.

\[ \xi_i(z) = \exp \left( -\frac{\|z - \mu_i\|^2}{\eta_i^2} \right) \quad (3) \]

where \( \mu_i = [\mu_{i1} \ldots \mu_{in}]^T \) and \( \eta_i \) are the center and width of Gaussian functions, respectively. By choosing enough nodes, NNs can approximate function \( h(.) \)

\[ h(z) = \Theta^* \xi(z) + \delta \quad (4) \]

where \( \delta \) is the approximation error of NN. The optimal weight vector \( \Theta^* \) is defined as

\[ \Theta^* = \arg \min \left\{ \sup \left| h(z) - \hat{h}(z) \right| \right\} \quad (5) \]

The pair (A,B) is controllable. This assumption assures that a gain matrix \( K_c \) exists such that the characteristic polynomial of \( A - BK_c^T \) is Hurwitz. This guarantees for a given positive definite matrix \( Q \), there exists a positive-definite solution \( P \) for the following matrix equation

\[ (A - BK_c^T)P + P(A - BK_c^T) + Q = 0 \quad (6) \]

STATE FEEDBACK STABILIZER DESIGN

In order to design state-feedback stabilizer, the control input is selected as

\[ u = -K_c^T x - u_{adp} - u_R - u_d \quad (7) \]

where \( u_{adp} \) is an adaptive part to compensate the effects of mismatched uncertainties. In addition, \( u_R \) and \( u_d \) are designed to compensate the effects of NN approximation error and external disturbance, respectively.
Define the functions $\eta^T$ and $g(x)$ as follows:

$$\eta^T(x) = x^T PB$$

(8)

$$g(x) = \frac{x^T P f(x)}{||\eta((x))||^2}$$

(9)

$$\dot{\hat{g}}(x) = \Theta \varepsilon(x)$$

(10)

The optimal parameter vector $\Theta^*$ is defined as

$$\Theta^* = \arg \min \left\{ \sup \left| g(x) - \hat{g}(x) \right| \right\}$$

(11)

Consider the adaptive neural controller, where

$$u_{adp} = \eta(x) \Theta \varepsilon(x)$$

(12)

$$u_R = k_1 \eta(x)$$

(13)

$$u_d = k_2 \text{sgn}(B^T P x)$$

(14)

The adaptation law for updating the estimation vector is

$$\dot{\Theta} = \gamma ||\eta||^2 \varepsilon^T(x)$$

(15)

**OBSERVER BASED STABILIZER DESIGN**

In a real dynamic system, the states of the systems may not be available for measurement. In this case, the results in Section III are not applicable in practice and the NN-based adaptive control using estimated states is then required. Let us consider the following mismatched uncertain system.

$$\dot{x} = Ax + f(x) + B[u + d]$$

(16)

$$y = C^T x$$

(17)

where $y$ are the measurable outputs of the system. The control objective is that the system states are regulated by using available outputs. Since the state vector $x$ is assumed to be immeasurable, it cannot be used in the controller design. Therefore, an observer should be designed to estimate the immeasurable states.

In order to design observer based stabilizer, the control input is selected as

$$u = -K_c^T \dot{x} - \eta(x) \Theta \varepsilon(\dot{x}) - u_a - u_R - u_d$$

(18)

$$u_a = K_0^T P \dot{x}$$

(19)

$$u_R = k_1 \eta(x)$$

(20)

$$u_d = k_2 \text{sgn}(B^T P x)$$

(21)

$u_a$ is a feedback of estimated states, $u_R$ and $u_d$ are the controllers to compensate NN approximation error and external disturbances, respectively. Fig.12 illustrates the states of the closed-loop system and corresponding estimated states. As seen in this figure, both states and estimated states are stabilized by using the observer-based controller.
The optimal parameter vector $\mathcal{O}^*$ is defined as

$$\mathcal{O}^* = \min[|g(x) - \hat{g}(x)|]$$  \hspace{1cm} (21)

Consider the adaptive neural controller, where

$$u_{adp} = \eta(x) \mathcal{O} \varepsilon(x)$$  \hspace{1cm} (22)

$$u_R = k_1 \eta(x)$$  \hspace{1cm} (23)

$$u_d = k_2 \text{sgn}(B^T P x)$$  \hspace{1cm} (24)

![Figure 1. Overall design procedure of the observer-based adaptive-neural stabilizer for mismatched uncertain systems.](image)

The adaptation law for updating the estimation vector $\mathcal{O}$ is

$$\dot{\mathcal{O}} = \gamma \|\eta\|^2 \varepsilon^T(x)$$  \hspace{1cm} (22)

**III. SIMULATION AND RESULTS**

In this section, the simulation study is carried out to show the proficiency of the proposed adaptive neural controller. Two numerical cases are considered to show the effectiveness of the proposed design. In addition, both state and observer-based controllers are presented for each case.

1) State-Feedback Design: The system considered here is a single-input nonlinear system obtained by adding a mismatched uncertain term to the system described in [28]. The resulting system can be written in terms of the following parameters:

$$A = \begin{bmatrix} -1.33 & -0.33 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad -1.25 x_1(t) + 0.072(1 - x_1(t)) \exp\left(\frac{x_2(t)}{1 + \frac{x_2(t)}{y_0}}\right)$$

$$f(x) = -1.55x_2(t) + 0.576(1 - x_1(t)) \exp\left(\frac{x_2(t)}{1 + \frac{x_2(t)}{y_0}}\right)$$

The goal is to investigate stabilization of this system with the state-feedback stabilizer proposed in this paper. The design parameters for the state-feedback stabilizer are given by
$K_f = \begin{bmatrix} 2.61 & 0.09 \end{bmatrix}$

Figure 2. State trajectories of the system by using state feedback adaptive-NN controller

Figure 3. Control input of the system by using state feedback adaptive-NN controller

Figure 4. Norm of weights of NN

Observer-Based Stabilizer Design

In this part, it is assumed that state vector $x$ is immeasurable. Therefore, an observer is to be designed for the estimation of the immeasurable states. Consider the system with the parameters $C_f$.

Suppose the output matrix $C_f$ is given as follows:

$C_f = \begin{bmatrix} 0 & 1 \end{bmatrix}$

Figure 5. True states and estimated states of the closed-loop system

Figure 6. Control signal of the system by using observer-based adaptive-NN controller
IV. CONCLUSION

Two state-feedback and observer-based adaptive neural network controllers for stabilization of nonlinear systems with mismatched uncertainties were presented. It was shown that the asymptotic convergence of the closed-loop system to zero is achieved while maintaining bounded states at the same time. The presented methods can handle both systems with \( n \leq 2m \) and \( n > 2m \), where \( n \) and \( m \) are the number of system states and control inputs, respectively. Simulation results reveal the effectiveness of the given methods in the stabilization of nonlinear systems with mismatched uncertainties. Future work includes real time implementation on hardware system.

REFERENCES