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Thermal Post buckling Analysis of Functionally Graded Materials Cylindrical Shell

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Abstract— This paper presents thermal post-buckling analysis of Functionally Graded materials cylindrical shell with temperature dependent and independent material properties. The basic formulation is based on higher order shear deformation theory (HSDT) with von-Karman nonlinear strain kinematics using modified C^0 continuity. A direct iterative based nonlinear finite element method (DIFEM) is used. The present outlined approach is validated with those available results in the literature.

Keywords- FGMs Cylindrical shell, Postbuckling, HSDT, Temperature dependent material property (TD) and Temperature independent material property (TID).

I. INTRODUCTION

Functionally graded materials (FGMs) have drawn considerable attention in many engineering applications since they were first proposed around 1984 in Japan. Typically, FGMs are made from a mixture of metals and ceramics. Advantages of FGMs over laminated composites are that material properties vary continuously and smoothly through the thickness from one surface to another surface. This is achieved by continuously varying volume fraction of constituent materials. These are high performance heat resistant material able to withstand ultra-high temperature and minimize thermal stress, while maintaining the desired structural integrity. Hence it can be used in many applications like plasma facing for nuclear reactor, wear resistant lining in the mineral processing industry, rocket heat shields and thermo-electric generators. For example, if the FGM is to be used to separate regions of high and low temperature, then at the hotter end it may consist of pure ceramic as the ceramic is having better resistance to higher temperatures. In contrast, the cooler end may be pure metal because of its better mechanical and heat transfer properties. In recent years the rapid advances in the manufacturing techniques of bulk FGMs have created exiting new possibilities of their applications in large scale structural system. A large number of literatures have been reported on post buckling analysis of cylindrical shell and plate. Lal et al. [1] studied post buckling response of functionally graded material plate subjected to mechanical and thermal loadings with random material properties based on higher order shear deformation theory. Yang et al. [2] presented the thermal post-buckling of FGMs and Shen [3] investigated on thermo-mechanical post-buckling of FGM cylindrical panels with temperature-dependent properties based on classical shell theory with von-Karman- Donnell-type nonlinearity. Singh et al. [4] investigated the C0 finite element for buckling of shear deformable laminated composite plates with random material properties.

II. FORMULATION

Consider a rectangular FGM cylindrical shell consisting of metal and ceramic at the top and bottom layer of length a, width b, and total thickness h, defined in(x, y, z) system with x- and -y axes located in the middle plane and its origin placed at the corner of the cylindrical shell.

Let $(\bar{u}, \bar{v}, \bar{w})$be the displacements parallel to the (x, y, z) axes, respectively as shown in Fig. 1.

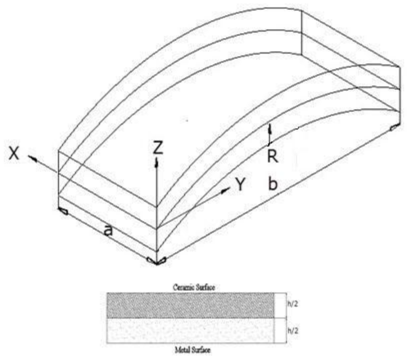

Figure 1.Geometry of the FGMs cylindrical shell.

The properties of the FGMs shell are assumed to be varying through the thickness according to power index law such that the top surface z=h/2 is ceramic reach and the bottom surface z=-h/2 is metal reach. The effective mechanical and thermal properties of the FGMs shell at an arbitrary point within the shell domain are expressed as,

$$V_c(z) = \left(0.5 + \frac{z}{h}\right)^n, \quad \frac{-h}{2} \leq z \leq \frac{h}{2}, \tag{1}$$

Where, n is volume fraction index and is always positive. For n=0, the shell is fully ceramic and when n=1, the composition of metal and ceramic is linear. The Poisson's ratio v depends weakly on temperature change and is assumed to be a constant

A. Displacement field model

Higher order shear deformation theory with C^0 continuity has been used to find displacement field model by using seven degrees of freedom

$$\bar{u} = u + f_1(z)\phi_x + f_2(z)\theta_x \quad \bar{v} = v(1+\frac{1}{R_1}) + f_1(z)\phi_y + f_2(z)\theta_y$$
$$\bar{w} = w \tag{2}$$

Where $(\bar{u}, \bar{v}, \bar{w})$ denote the displacement of a point along the (X, Y, Z) coordinates axes, (u, v, w) are corresponding displacements of a point on the mid plane, ϕ_x and ϕ_y are the rotations at Z=0 of normal to the mid surface with respect to X and Y axes, θ_x and θ_y are the slopes along X and Y directions $\theta_x = \frac{dw}{dx}$ and $\theta_y = \frac{dw}{dy}$. The function $f_1(z)$ and $f_2(z)$ can be written as,

$$f_1(z) = C_1 z - C_2 z^3; f_2(z) = -C_4 z^3, \text{ With } C_1=1,\ C_2 = C_4 = \frac{4h^2}{3} \tag{3}$$

The displacement vector for the modified model can be written as,

$$\Lambda = [u\ v\ w\ \theta_y \theta_x\ \phi_y \phi_x]^T \tag{4}$$

B. Governing Equation

Using the finite element method, the governing equation for buckling load of the cylindrical shell analysis can be derived using the variational principle

$$[K_l + K_{nl}\{q\}]\{q\} = [F^T] \tag{5}$$

Substituting the values and obtaining in the form of nonlinear generalized eigenvalue problem as,

$$[K]\{q\} = \lambda[Kg]\{q\} \tag{6}$$

Where $\{q\}$, $[Kg]$ and λ are defined as a global displacement vector, global stiffness and geometric stiffness matrices, global mass matrix and critical buckling temperature respectively.

III. RESULT AND DISCUSSION

A DIFEM approach outline is presented in the previous subsection for post buckling analysis of the FGM cylindrical shell subjected to thermal loading with temperature dependent and independent material properties. A nine nodes Lagrange isoparametric element with 63 DOFs per element in the present HSDT model has been used. Based on convergence study the (4 × 4) mesh has been used in the present paper. In the present study following boundary conditions are applied,

All edges simply supported (SSSS):

$$v = w = \theta_y = \psi_y = 0, \; at \; x = 0, a$$

$$u = w = \theta_x = \psi_x = 0 \; at \; y = 0, b$$

All edges clamped (CCCC):

$$u = v = w = \theta_x = \theta_y = \psi_x = \psi_y = 0, \; at \; x = 0, a \quad \text{And} \quad y = 0, b.$$

Typical results are presented for functionally graded cylindrical shell with various volume fractions index and SSSS, CCCC boundary conditions. The dimensionless mean buckling load of the FGM cylindrical shell subjected to thermal (λ_{Tcr}) loading is defined as,

$$\lambda_{Tcr} = \alpha_0 \Delta T_{cr} \; x 10^3$$

Table 1. The following TD and TID material properties of Silicon nitride/SUS304 used for computation [1]

Types of Material	Properties	P-1	P0	P1	P2	P3
Silicon nitride	E (Pa)	0	348.43e9	-3.070e-4	2.160e-7	-8.946e-11
	v	0	0.24	0	0	0
	α (1/k)	0	5.8723e-6	9.095e-4	0	0
SUS304	E (Pa)	0	201.04e9	3.079-4	-6.534e-9	0
	v	0	0.3262	0	0	0
	α (1/k)	0	12.330e-6	8.705e-4	0	0

3.1 Validation study for dimensionless buckling load

The dimensionless mean post-buckling load of FGMs (Si3N4/ SUS304) cylindrical shell with CCCC support condition having a/h=100 is shown in table 2. It is seen from the table that present results of thermal buckling load obtained by HSDT based on finite element method are in very good agreement with the available semi analytical method published results of Yang J et. al. [2].

Table 2.Validation table for thickness ratio, volume fraction index and dimensionless bucking temperature parameter

$$\lambda_{Tcr} = \alpha_0 \Delta T_{cr} \text{ x } 10^3$$

n	TD		TID	
	Present	Yang et al. [2]	Present	Yang et al. [2]
0	2.9705	3.1742	2.8606	3.7492
0.5	2.5500	2.5941	2.4763	2.9739
2	2.0485	2.3209	2.2243	2.6174
10	1.6919	2.2018	1.8301	2.4628
Metal	1.7477	2.0284	1.4839	2.2554

3.2 Convergence and comparison study

The accuracy and effectiveness of present DIFEM method is shown in Table. 3, it is observed from Table. 3 as 4X4 Mesh size is good converged; therefore it is used in the analysis.

Table 3.Convergence study for dimensionless thermal Post buckling load of SUS304/Si3N4 cylindrical shell with clamped supported edges, $\lambda_{Tcr} = \alpha_0 \Delta T_{cr} \text{ x } 10^3$ ***,a=0.6, b=0.3, a/h=100***

Mesh Size	Volume fraction Index (n)				
	0	0.5	2	10	Metal
3 x 3	14.914	12.85	10.271	8.4399	7.7592
4 x 4	2.9705	2.55	2.0485	1.6919	1.7477
5 x 5	1.3147	1.184	1.0776	0.9028	0.8223
6 x 6	1.2198	1.005	0.8142	0.7026	0.6127
Yang et al. [2]	3.1742	2.594	2.3209	2.2018	2.0284
% Difference	6.40%	1.70%	11.70%	23%	13%

3.3 Parametric Study

The effect of all edges clamped support boundary condition is more desirable than the all edges simply supported in FGMs cylindrical shell and it is observed that shell with TD material properties shows higher buckling load as compared to the TID material properties. It can also be seen volume fraction index increases buckling load goes decreases shown in Table. 4. This due to the increase presence of metal containment in FGM shell which is affecting on stiffness matrix.

Table 4.Uniform temperature distribution having TD and TID properties on the post buckling analysis of FGMs cylindrical shell subjected to thermal loading (a/h=100, a=0.6, b=0.3, R=1, b/R=0, $\lambda_{Tcr} = \alpha_0 \Delta T_{cr} \times 10^3$ **.**

Boundary condition	Temperature Distribution in K	n	Uniform Temperature Distribution	
			TD	TID
CCCC	100	0	2.9705 (1.9283)	2.8606 (1.8690)
		5	1.8028 (1.1771)	1.9515 (1.1181)
		10	1.6919 (1.1038)	1.8301 (1.0482)
SSSS		0	2.6932 (1.6271)	2.8879 (1.5461)
		5	1.8550 (0.9956)	1.5562 (0.9399)
		10	1.5342 (0.9338)	1.4640 (0.8842)

CONCLUSIONS

The DIFEM procedure based on HSDT is presented to study of post-buckling analysis of FGMs cylindrical shell subjected to thermal loading with TD and TID material properties. The effects of volume fraction index, boundary conditions and different temperature distribution have been studied in this paper. The following conclusions can draw from the limited study.

- It is seen that as volume fraction index increases buckling load of FGMs cylindrical shell decreases. Due to the more metal present in composition.
- The FGMs cylindrical shell with all the edges clamped (CCCC) support condition shows the higher post buckling load than the SSSS support condition. This is due to the increase in the boundary constrains.

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