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Exact Through Thickness Temperature Variation of Laminated Beam

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Abstract— Exact through thickness temperature variation in the two dimensional (2D) laminates have been obtained using semi-analytical approach developed by Kant *et al.* [1]. The mathematical model is developed with the help of Fourier's law and partial differential equation (PDE) of heat conduction and its consists of defining two point boundary value problem (BVP) governed by a set of coupled first-order ordinary differential equations (ODEs). Fourth order Runga-Kutta-Gill (RK-Gill4) method is used for numerical integration. The accuracy and the effectiveness of the developed model is assessed by comparing obtained numerical results from the present work with the available exact solution.

Keywords- Semi-analytical, Two point BVP, RK-Gill4, Thermal load

I. INTRODUCTION

Composite and sandwich laminates are extensively used in the aerospace structures. These materials are also used in field of civil structures, biomedical and electronic devices due to its superficial advantages. Enhanced temperature field is the major factor for the failure of these materials. Due to thermal load, delamination of layers and the longitudinal cracks occurs in the laminates and therefore, it is important to obtain the actual temperature variation through thickness of the laminates.

Most of the researches reported in the literature have been assumed linear, constant or exponential temperature variation through thickness of the laminates. Tungikar and Rao [2] obtained layer wise temperature profile for a thick anisotropic laminate which proves the variation of temperature not remains linear through the thickness of domain. Carrera [3] studied the errors incorporated in the anisotropic plate due to the assumption of linear temperature variation through the thickness of plate. Actual through thickness temperature variation is obtained by Kapuria *et al*. [4], [5] and [6] for various types of orthotropic composite and sandwich laminates. This all research shows that the assumed temperature variation through the thickness of domain in stress analysis is not correct and need to be capture accurately at least for thick domain.

An effort has been put in this work to developed semi-analytical model by using Fourier's law and partial differential equation (PDE) of heat conduction to capture exact variation of temperature field through the thickness of laminated beam. The mathematical model consist of defining two-point BVP governed by a set of coupled first-order ODEs,

$$\frac{d}{dz}y(z) = B(z)y(z) + p(z) \tag{1}$$

within the domain $-h/2 \leq z \leq +h/2$.

II. MATHEMATICAL FORMULATION

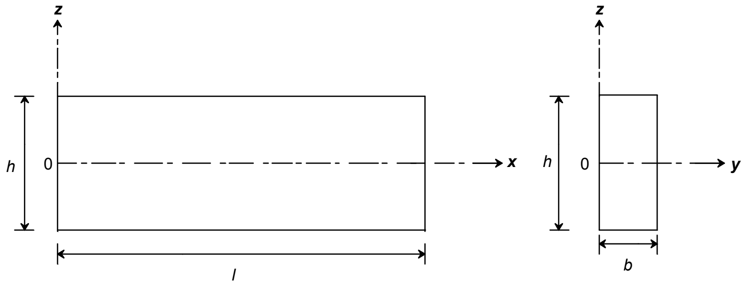

Figure 1. Laminated beam subjected to thermal load

A layered, narrow, simply supported beam is subjected to thermal load varies along length 'l' with no variation along width 'b' is considered (***Figure 1***). The thickness of beam is 'h' composed of number of isotropic and/or orthotropic laminates perfectly bonded together. By imposing the plane-stress conditions of elasticity, the dimension is reduced from three dimensional (3D) to two dimensional (2D) one.

According to Fourier's law of heat conduction, heat flux in direction x and z is given by

$$q_x = -\lambda_1 \frac{\partial T}{\partial x} \qquad q_z = -\lambda_3 \frac{\partial T}{\partial z} \tag{2}$$

Where, λ_i = coefficient of thermal conductivity along x and z axis (i=1, 2, 3) in $Wm^{-1}K^{-1}$

q_i = heat flux along x and z axis ($i=x, z$) in Wm^{-2}

T = temperature in Kelvin (K)

With the assumption that amount of heat remain in the element due to heat flow is zero, the equilibrium equation in 2D,

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_z}{\partial z} = 0 \tag{3}$$

For 2D domain, the steady state heat conduction equation for the homogeneous isotropic/orthotropic material without internal heat generation is

$$\lambda_1 \frac{\partial^2 T}{\partial x^2} + \lambda_3 \frac{\partial^2 T}{\partial z^2} = 0 \tag{4}$$

For laminated components, continuity of temperature and heat flux have to be maintained at the interface for accurate analysis. This conditions have been satisfied in the developed formulation. By using algebraic manipulation of the Equations (2), (3) and (4), a set of PDEs involving only two dependent variables T and q_z are obtained as follows.

$$\frac{\partial T}{\partial z} = -\frac{1}{\lambda_3} q_z \qquad \text{and} \qquad \frac{\partial q_z}{\partial z} = \lambda_1 \frac{\partial^2 T}{\partial x^2}$$

(5)

The above PDEs defined by Equation (5) can be reduced to a coupled first-order ODEs by using Fourier trigonometric series expansion for primary variables satisfying the simply supported boundary conditions at x=0 and a, as follows,

$$T = \sum_{m=1}^{\infty} T(z) \sin\frac{m\pi x}{l} \qquad \text{and} \qquad q_z = \sum_{m=1}^{\infty} q_z(z) \sin\frac{m\pi x}{l} \tag{6}$$

Substituting Equation (6) and its derivatives into Equation (5), the following set of first-order ODEs is obtained

$$\frac{dT(z)}{dz} = -\frac{1}{\lambda_3} q_z(z) \quad \text{and} \quad \frac{dq_z(z)}{dz} = -\lambda_1 \frac{m^2\pi^2}{l^2} T(z) \tag{7}$$

Equation (7) represents the governing two point BVP in ODEs in domain $-h/2 \leq z \leq +h/2$ with known temperatures at the top and bottom surface of a beam. Changes in material properties are incorporated by changing the coefficients of material matrix appropriately for each lamina.

III. NUMERICAL STUDY

The accuracy and effectiveness of present formulation is assessed by comparing present numerical results with the available exact solution presented by Kapuria *et al.* [4]. Solution is obtained by Runge-Kutta-Gill numerical integration technique. The thermal analysis of beams have been done for two thermal load conditions.

1. Equal temperature rise of the bottom and the top faces of the beam with sinusoidal longitudinal variation: (Load case 1)

$$T(x, \pm h/2) = T(z)\sin\frac{m\pi x}{l}$$

2. Equal rise and fall of temperature of the top and bottom faces of the beam with sinusoidal longitudinal variation: (Load case 2)

$$T(x, +h/2) = -T(x, -h/2) = T(z)\sin\frac{m\pi x}{l}$$

Aspect ratio considered is (l/h) = 5. Material properties used for the numerical study are tabulated in ***Table 1***. Convergence studies have been performed to fix the required number of steps for numerical integration and it is observed that around 20-30 steps are sufficient to get converged solution.

Problem 1. Five layered composite beam with laminae thickness are 0.1h, 0.25h, 0.15h, 0.2h and 0.3h of material sets 1/2/3/1/3 (***Table 1***) which are highly inhomogeneous coefficients of thermal expansion and thermal conductivities has been considered. The stacking sequence is maintained from the bottom of laminate. Orientation of fibers in all laminates are 0^0 measured with reference to x axis. The comparison of the present numerical solutions with elasticity solution presented by Kapuria *et al.* [4] have been depicted in ***Figure 2***. It is observed that through thickness variation obtained by present formulation are in well agreement with elasticity solution. Due to inhomogeneous material properties, non-linear through thickness temperature variation have been observed in load case 1 where as a linear temperature variation through the thickness of laminated beam have been observed in load case 2.

Problem 2. A symmetric, three layered, sandwich beam with laminae thickness are 0.1h, 0.8h and 0.1h of material sets face sheet/core sheet/face sheet (***Table 1***), where face sheet has graphite-epoxy material and core has soft material. Orientation of fibers in all laminates are 0^0 measured with respect to reference axis x. The comparison of the present numerical solutions with elasticity solution presented by Kapuria *et al.* [4] have been depicted in ***Figure 3*** and exact match with elasticity solution again proves the accuracy of the present development. For load case 1, temperature variation in face sheets observed to be linear whereas parabolic variation is observed in soft core region. For load case 2, linear variation is observed for all three layers.

IV. CONCLUDING REMARK

The developed semi-analytical mathematical model is simple, efficient, highly accurate and free from any simplified assumptions through the thickness of beam laminate. The results obtained by present formulation are exactly matching with the elasticity solution which helps to prove the accuracy of the present development. The actual temperature variation in the laminates showed that assumed linear, constant or exponential temperature variation through thickness of the laminates are not correct.

Hence, exact variation of temperature need to be consider in the stress analysis to capture laminate behavior under thermal loading condition accurately.

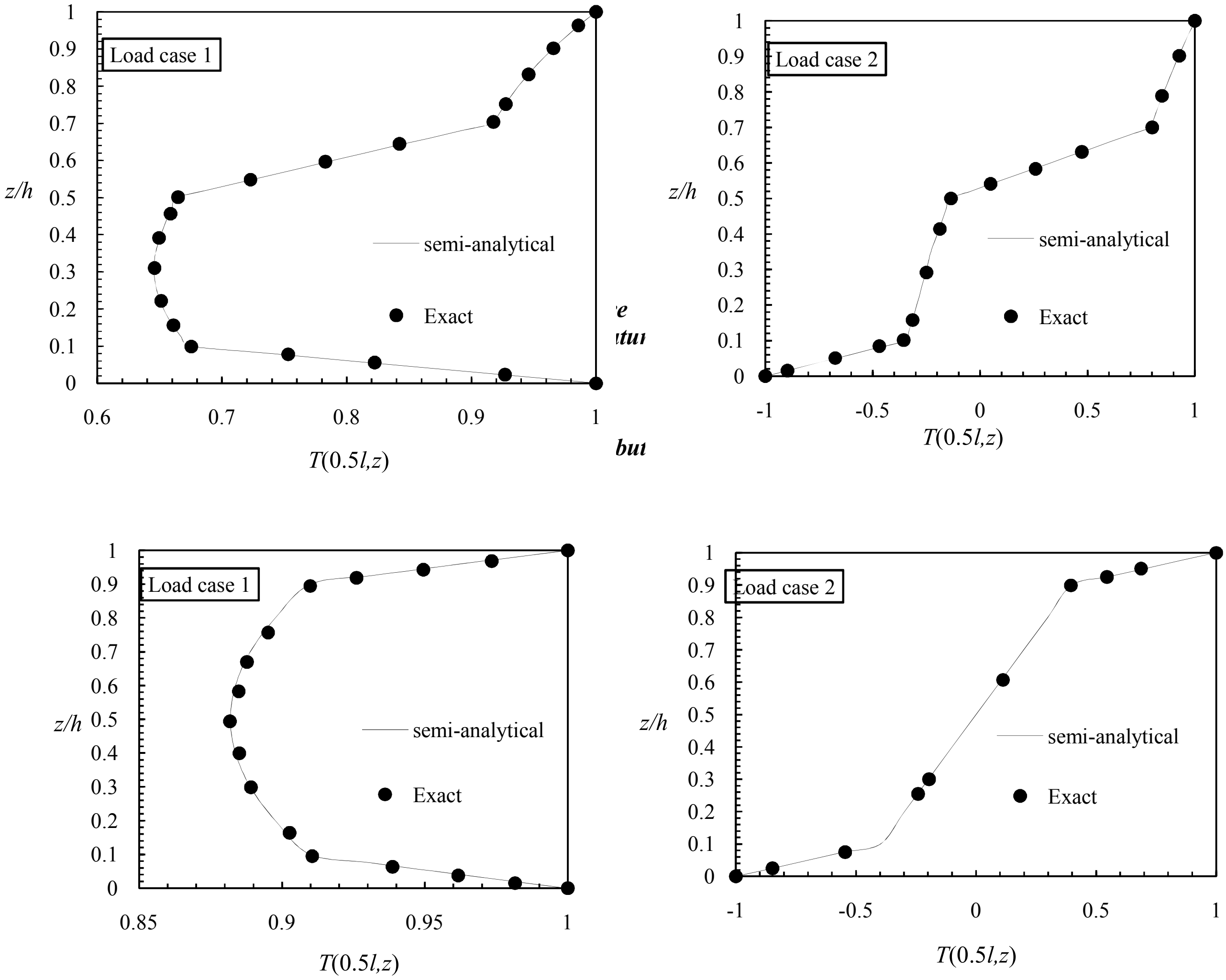

Figure3. Temperature distribution for sandwich beam under load case 1and 2

Table 1. Material properties

Properties	Material 1	Material 2	Material 3	Material 4	Face sheet	Core sheet
E_1	6.9	224.25	172.5	131.1	131.1	0.2208
E_2	6.9	6.9	6.9	6.9	6.9	0.2001
E_3	6.9	6.9	6.9	6.9	6.9	2760
ν_{12}	0.25	0.25	0.25	0.32	0.32	0.99
ν_{13}	0.25	0.25	0.25	0.32	0.32	3E-5
ν_{23}	0.25	0.25	0.25	0.32	0.32	3E-5
G_{12}	1.38	56.58	3.45	3.588	3.588	16.56
G_{13}	1.38	56.58	3.45	3.588	3.588	545.1
G_{23}	1.38	1.38	1.38	2.332	2.332	455.4
α_1	35.6E-6	0.25E-6	0.57E-6	0.02E-6	0.023E-6	30.6E-6
α_2	35.6E-6	35.6E-6	35.6E-6	22.5E-6	22.5E-6	30.6E-6

α_3	35.6E-6	35.6E-6	35.6E-6	22.5E-6	22.5E-6	30.6E-6
λ_1	0.12	7.2	1.92	1.5	1.5	3
λ_2	0.12	1.44	0.96	0.5	0.5	3
λ_3	0.12	1.44	0.96	0.5	0.5	3

Reference: Kapuria *et al.* [4]

Young's modulus (E), shear modulus (G) in the GPa, thermal coefficient of expansion (α) in K^{-1}, coefficient of thermal conductivity (λ) in $Wm^{-1}K^{-1}$

REFERENCE

[1] T. Kant, S. S. Pendhari and Y. M. Desai, "An Efficient Semi-Analytical Model for Composite and Sandwich Plates Subjected to Thermal Load", Journal of Thermal Stresses, vol. 31, pp. 77–103, 2008.

[2] V. B. Tungikar and K. M. Rao, "Three Dimensional Exact Solution of Thermal Stresses in Rectangular Composite Laminate", Composite Structures, vol. 27, pp. 419-430, 1994.

[3] E. Carrera, "Temperature Profile Influences on Layered Plates Response Considering Classical and Advanced Theories", American Institute of Aeronautics and Astronautics (AIAA), vol. 40, pp.1885–1896, 2002.

[4] S. Kapuria, P.C. Dumir and A. Ahmed, "An Efficient Higher Order Zigzag Theory for Composite and Sandwich Beams Subjected to Thermal Loading", International Journal of Solids and Structures, vol. 40, pp. 6613-6631, 2003.

[5] S. Kapuria and G.G.S. Achary, "An Efficient Higher Order Zigzag Theory for Laminated Plates Subjected to Thermal Loading," International Journal of Solids and Structures, vol. 41, pp. 4661-4684, 2004.

[6] S. Kapuria and J. K. Nath, "Improved Global-Local Theory for Laminated Plates Under Thermal Load with Actual Temperature Profile", Journal of Thermal Stresses, vol. 35, pp. 169–191, 2012.

