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Multiple Primary User Spectrum Sensing Using John's Detector at Low SNR

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Abstract— In multi-antenna cooperative spectrum sensing in cognitive radio, there are multiple primary users and multipath channels. In low signal to noise regime noise-uncertainty-free detector is optimal; we are using test statistics which help simple accurate analytical formulae. False alarm, detection probabilities and receiver operating characteristics are given in closed form. Here we observe simulation settings, performance gain over several detection algorithm observed in low signal to noise ratio.

Keywords- cognitive radio, locally best invariant test, moment-based approximation, multipath channels, multiple primary users, spectrum sensing.

I. INTRODUCTION

Cognitive radio networks useful for dynamic spectrum access which help to reduce spectrum scarcity, also CRs not allowed intolerable interference to the licensed users. Spectrum sensing is important step for dynamic spectrum access area. Single active primary user with one antenna, so multiple primary users will have multi-antenna in the spectrum with each one has single- tap channel, Based on this several eigenvalue based have proposed [1-8].Single-tap channel may fail to reflect in highly frequency selective channels. Single primary user may not be available in forthcoming CR networks, In cellular network more than one primary users present commonly, In Unlicensed bands such as Bluetooth, Wi-Fi, may share same band without co-ordination resulting the multiple primary user system [9].Exiting primary user detection algorithm will decrease the performance, because of that we need to understand multi-primary user detection with multiple sensors, so we are working on same scenarios. A ST (Spherical test) based detection algorithm has been proposed for multi-primary sensing of spectrum in [10], also studied in [11]. ST detector is best known primary user detector, but it has limitation for low Signal-to-Noise Ratio (SNR) regime. Spherical test detector does not perform particularly well when SNR is low [11,12].At the low SNR spectrum sensing is practically channeling issue, FCC (Federal Communication Commission) require that secondary devise must be able to detect signal with low SNR as -18dB [13,14] , Up to this there is no optical low SNR spectrum sensing algorithm for multiple primary users.
In this paper we address the problem for multiple primary users spectrum sensing and we have to overcome the problem of low SNR regime and we are analyzing how spectrum sense at low SNR, and to help to regards to the same we have develop the test statics analysis of low SNR detector, Using the test statics analysis approximation to the detection probability, false alarm probability and ROC(Receiver Operating Characteristics) are developed and proposed approximation easily computable and reasonably accurate. Our proposed algorithm shows effectiveness of multiple

primary user detectors analyzed in four propositions. The whole paper is organized as follows, In Section II we studied the problem of multi-sensor spectrum sensing in multiple primary area, Section III gives details about multiple primary user sensing in low SNR, and the numerical examples gives required result in Section IV, Section V will give main result of the paper.

II. PROBLEM FORMULATION

We analyzing when there are multiple primary users i.e. ($P \geq 1$) within bandwidth W, so transmission sample interval is an integer multiple of minimum duration $T \leq 1/W$. For TDM (Time division multiplexing) the sample duration is symbol interval, CDM (Code-division multiplexing) primaries are the chip duration and OFDM (Orthogonal Frequency Division Multiplexing) is the inverse Fourier bandwidth, Minimum bandwidth of transmission of primaries within W is sample duration S_{max} T. In this there is detector with K sensors trying to sense the primary user's .Channels between detector and PUs may be multipath, and transmission may be or may not be asynchronous. The multipath structure of primary users not know to the detector, All the sample duration between T and S_{max} T will be use by PU, and this is known to the detector, the sample time at the transmitter and receiver are subject to negligible clock flow.

A. Signal Model

The receive baseband signal after the filtering at the K sensors in bandwidth interest is

$$x(t)=\sum_{m} \sum_{p=1}^{p} s_{p,m} h_p\left(t - mT_p + d_p\right) + n(t) \qquad 2.1$$

Tp=spT is the sample duration for transmission p, hp(t) is the convolution of the reception filter and also vector channel response between p and K sensors. Sum over m is the transmission samples. dp is showing the differences in symbol timing and propagation delay for different transmitters. Sensor K detector select ss to be smallest common multiple of possible values of sp, take sample of every ssT seconds. Assuming proper receiver filtering, the K × 1 vector n represents additive complex Gaussian noise with mean zero and covariance matrix σ2IK, where σ2 is the noise power, where □p :E[spspT] is the signal power of Primary user p transmission. We collect N observations to a K ×N ($K \leq N$) received data matrix X = [x1. . . xN] = HS, where S = [s1, . . . , sN]. We assume that the transmitted samples sp,m are independent and follow a standard complex Gaussian distribution. They are by definition independent from the noise. This holds for TDM, pseudo randomly spread CDM, and approximately for OFDM with a large number of subcarriers. We further assume that the channel H is constant during sensing i.e. deterministic channels, and is subject to negligible inaccuracies arising from clock drifts. Thus a channel model needs not be specified to carry out the analysis in this paper. Our focus is performance analysis for given channel realizations.

B. Sensing problem

The problem of interest is to use the data matrix X to decide whether there are primary users. By the considering in the last subsection, the sample covariance matrix $R = XX^{\dagger}$ of the received data matrix follows a complex Wishart distribution, represented by $R \sim W_K$ (N, Σ) and population covariance matrix calculated in the absence of primary users, it is denoted by hypothesis H_0, is

$$H_0 : \Sigma := E[XX^T]/N = \sigma^2 I_K \qquad 2.2$$

And hypothesis H_1 represents the presence of primary users,

$$H_1 : \ \Sigma = \sigma^2 I_K + \sum_{p=1}^{p} \gamma_p h_p h_p^{\,T} \qquad 2.3$$

The SNR at the receiving of primary user p across the K sensors is $SNR_p := \gamma_p \|h_p\|^2/\sigma^2$.The interference level near to the primary transmitter from the secondary system transmission, the dynamic spectrum management control all this. The differences between the population covariance matrices can be explored to detect the primary users. Wrongly declaration of H0, or correctly declaration H1, denote the false alarm probability Pfa, and the detection probability Pd, respectively. The sample covariance matrix **R** is a Wishart matrix, it is sufficient statistics for the population covariance matrix **Σ** [15], and on the knowledge of the noise power σ^2, different test statistics can be derived as functions of **R**. When multiple primary users (P ≥ 1) but not known a priori information, the matrix $\sum_{p=1}^{p} \gamma_p h_p h_p^T$is positive definite. Thus, the notation $A \succ B$ is equivalent with A–B positive, the hypothesis is

$$H_0: \Sigma = \sigma^2 I_K \quad 2.4$$

$$H_1: \Sigma > \sigma^2 I_K \quad 2.5$$

where, we assuming the noise power σ2 to be unknown. Essentially, we are testing a null hypothesis Σ = σ2IK against all the other possible alternatives of Σ, so we can say the hypothesis testis unknown to P. The GLR-optimal detector under the hypotheses test is based on the so-called Spherical Test (ST) [10,11].ST detector giving good result but it is not best detector in low SNR, for multiple PUs, test statistics is good in small derivation form H0 is based on

$$T_J = \frac{tr(R^2)}{(tr(R)^2)} = \frac{\sum_{i=1}^{K}(\lambda_i)^2}{(\sum_{i=1}^{K}\lambda_i)^2} \quad 2.6$$

S.John presented test statistics in[16],the resulting test procedure is

$$T_j \underset{H0}{\overset{H1}{\gtrless}} \zeta \quad 2.7$$

ζ is a threshold, John detector is known as Locally Best Invariants [17], for every σ^2 and test T, there is neighborhood of $\sigma^2 I_K$ such that T_j has no bad performance than T [17].

III. PERFORMANCE ANALYSIS

We derive closed-form expressions for the moments of T_J under both hypotheses. Based on the derived results, we construct approximations to the distributions of T_J, which lead to analytical formulae for the false alarm probability, the detection probability, as well as the receiver operating characteristic.

A .False Alarm Probability

The m-th non-negative integer moment variable T_J under H_0 is

$$\mu_m := \frac{C\Gamma(KN)}{\Gamma(2m+KN)} \times \sum_{a_1+\cdots a_K=m} \frac{m!}{a_1!\ldots a_K!} \times \prod_{1\le i<j\le K}(2a_j - 2a_i + j - i) \prod_{i=1}^{K}\Gamma(2a_j + N - K + i) \quad 3.1$$

Where **Γ**(.) defines the gamma function constant $C = (\prod_{i=1}^{K}\Gamma(N-i+1)\ \Gamma(N-i+1))^{-1}$.
For any sensor size K and sample size N, the Beta approximation to the CDF of T_J under H_0, based on the exact two first moments in

$$F_J(y) \approx 1 - B\left(\frac{\frac{K(1-y)}{K-1};\beta_0,\alpha_0}{B(\alpha_0,\beta_0)}\right),\ y \in [1/K,1] \quad 3.2$$

Where

$$\alpha_0 = \frac{(K\mu_1 - 1)(K\mu_1 - K\mu_2 + \mu_1 - 1)}{(K-1)K(\mu_2 - \mu_1^2)} \qquad 3.2.1$$

$$\beta_0 = \frac{(\mu_1 - 1)(K\mu_1 - K\mu_2 + \mu_1 - 1)}{(K-1)K(\mu_1^2 - \mu_2)} \qquad 3.2.2$$

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \qquad 3.3$$

$$B(x;a,b) = \int_0^x z^{a-1}(1-z)^{b-1}\,dz \qquad 3.4$$

Both 3.3.1 and 3.3.2 defines the Beta function and the lower incomplete Beta function respectively. The false alarm probability, for a given threshold ζ, equals
The P_{fa} requirement a threshold can be calculated by numerically inverting $F_J(\zeta)$,

$$\zeta = F^{-1} J\,(1 - Pfa) \qquad 3.5$$

B. Detection probability

We first study the moments of T_J under H_1. For convenience of the discussion we define the random variables

$$x := 1/N^2\, tr(R^2), \quad y := 1/N\, tr(R), \quad z := x/y^2. \qquad 3.6$$

Clearly, *z* is the random variable of interest, T_J. Unlike the case of H_0, the equality no longer holds under H_1. In order to estimate the moments of *z* it is not enough to estimate the moments of random variables *x* and *y* separately, estimating their correlation is needed. A standard technique in this situation is the so-called 'Delta method which relies on the Taylor expansions for the moments of random variable *z*.

$$\mu_z \approx \frac{\mu_x}{{\mu_y}^2} - \frac{2\mu_{xy}}{{\mu_y}^3} + \frac{3\mu_x \nu_y}{{\mu_y}^4} \qquad 3.7$$

$$\nu_z \approx \frac{\nu_x}{{\mu_y}^4} - \frac{4\mu_x \mu_{xy}}{{\mu_y}^5} + \frac{3\mu_x \nu_y}{{\mu_y}^6} \qquad 3.8$$

μ_{xy} denotes the covariance of x and y. Here the quantities μ_x, μ_y, ν_x, ν_y and μ_{xy} are calculated

$$\mu_x = tr(\Sigma^2) + \tfrac{1}{N}(tr(\Sigma))^2 \qquad 3.8.1$$

$$\nu_x = \frac{4}{N} tr(\Sigma^4) + \frac{2}{N^2}(4tr(\Sigma)tr(\Sigma^3) + (tr(\Sigma^2))^2 \frac{2}{N^3}(2(tr(\Sigma))^2 tr(\Sigma^2) + \left(tr(\Sigma^2)\right)^2 + tr(\Sigma^4) \qquad 3.8.2$$

$$\mu_y = tr(\Sigma) \qquad 3.8.3$$

$$v_y = \frac{1}{N} tr(\Sigma^2) \qquad 3.8.4$$

$$\mu_{xy} = \frac{2}{N} tr(\Sigma^3) + \frac{2}{N^2} tr(\Sigma) tr(\Sigma^2) \qquad 3.8.5$$

With the estimates of the mean and variance, closed-form distributions of T_J under H_1 can be constructed. Here we also choose the Beta distribution, since it has the same support as T_J. An additional motivation comes form the fact that for K = 2 the test statistics T_J becomes a linear transform of the test statistics T_{ST}, whose exact distribution under H_1 holds the same polynomial form as the Beta distribution.

$$G_J(y) = 1 - \left(\frac{B\left(\frac{K(1-y)}{K-1}; \beta_1, \alpha_1\right)}{B(\alpha_1, \beta_1)} \right), \quad y£ [1/K,1] \qquad 3.9$$

$$\alpha_1 = \frac{(1-K\mu_z)((\mu_z-1)(K\mu_z-1)+Kv_z}{(K-1)\mu_z} \qquad 3.10$$

$$\beta_1 = \frac{(\mu_z-1)((\mu_z-1)(K\mu_z-1)+Kv_z}{(K-1)\mu_z} \qquad 3.11$$

The resulting approximation to the detection probability reads

$$P_d(\zeta) = 1 - G_J(\zeta) \approx \frac{B\left(\frac{K(1-\zeta)}{K-1}; \beta_1, \alpha_1\right)}{B(\alpha_1, \beta_1)} \qquad 3.12$$

Where, $\zeta \in [1/K, 1]$.

The mapping between the false alarm probability and the detection probability is referred to as the receiver operating characteristic.

$$P_d = 1 - G_J(F_J^{-1}(1-P_{fa})) \qquad 3.13$$

Parameters α_0 ,β_0 and α_1 ,β_1 are the only elementary function of the sensor size K, N is sample size and $\sum$ is the population covariance matrix, if values of this parameters to their respective integer, both false alarm probability and detection probability reduce to finite polynomials in the threshold ζ, so computation simple for on-line implementation. The exact false alarm probability and detection probability can be written as a sum of the n-moment-based approximation.

IV. RESULT

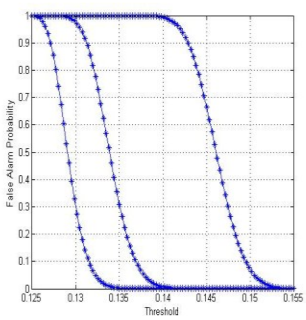

Figure 1

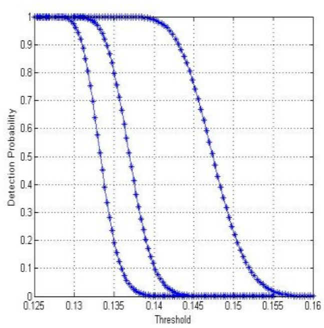

Figure 2

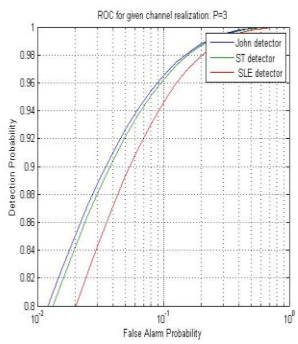

Figure 3

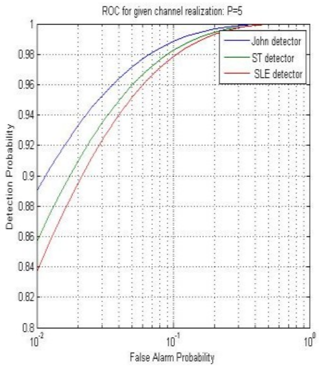

Figure 4

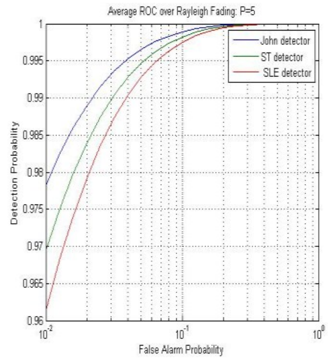

Figure 5

Figure 1 is the plot of false alarm probability as function of threshold, Figure 1 and Figure 2 uniform sampling in ζ € [.0125,0.2] with sampling size 10^6, Figure 1 shows analytical analysis false alarm matches with simulation and False alarm probability for K=10, N=50,100,and 200. In Figure2 we have plot detection probability is function of threshold and we have selected three primary user (P=3) having SNR_1= -4dB, SNR_2=-5dB, SNR_3= -6dB.Figure 3 three primary users (p=3) having SNR_1=-4dB,SNR_2=-5dB,SNR_3=-6dB and we have chosen number sensors K=4 and number of sample per sensor is N=200 having eigenvalues ∑ are [1.54,1.25,1.02,1], In this the ST detector and Johns detector having closed performance not to low SNR, the performance of SLE detector degraded for multiple primary users, so SLE detector not optimal for multiple primary users.

In Figure 4 we are chosen five primary users (P=5) having SNR_1=0dB,SNR_2=-2dB,SNR_3=-4dB,SNR_4=-6dB and SNR_5=-8dB using K=4 sensors and N=100 sample per sensor, we assume channel matrix is independently drawn from a standard complex Gaussian distribution and kept constant during sensing. Figure 5 shows average detection performance over fading channel via simulation, up to this our ROC giving fixed ∑ ,In Rayleigh fading we plot ROC curve over 10^4 realization of ∑, number of detector same as Figure 3 and 4 K=4, Number of sample N=100 and number of primary users five (P=5) having SNR_1=1dB, SNR_2=-3dB, SNR_3=-7dB, SNR_4=-11dB, SNR_5=-15dB.

V. CONCLUSION

So we have investigated the performance of Johns detector, which is useful for multiple primary user detector at low SNR compare with ST detector and SLE detector, we have resented analytical formulas for false alarm probability, detection probability and ROC, also compare with simulation result. We have compare detection algorithms for single primary user and detection algorithm for multiple primary users, In that SLE is useful for single primary user, ST detector for multiple primary users but not for low SNR and Johns detector for multiple primary users at low SNR, So John's detector useful for multiple primary spectrum sensing at low SNR.

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